

Algorithms & Complexity

Non-Deterministic Space

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Non-Deterministic *SPACE*

- ▶ For nondeterministic Turing machine the space complexity is denoted *NSPACE*.
- ▶ $NSPACE(T, x)$ is the number of cells used by the *non-deterministic* computation $T(x)$. We will say that $NSPACE(T) = O(f(n))$ if $NSPACE(T, x) = O(f(n))$ with $n = |x|$ (length of x).
- ▶ $NSPACE(f(n)) = \{A | \exists T \text{ s.t. } T \text{ solves } A \text{ and } NSPACE(T) = O(f(n))\}$
- ▶ $DSPACE(f(n)) \subseteq NSPACE(f(n))$

Non-Deterministic *SPACE* complexity classes

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- ▶ $NPSPACE = \cup_{k \in \mathbb{N}} NSPACE(n^k)$

Savitch's Theorem

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For a function $f(n) \geq \log(n)$:

$$NSPACE(f(n)) \subseteq DSPACE((f(n))^2).$$

Savitch's Theorem: a Corollary

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- ▶ $\text{NPSPACE} = \text{PSPACE}$
- ▶ This follows directly from the fact that the square of a polynomial function is still a polynomial function.

Immerman-Szelepcsényi Theorem

- ▶ Immerman-Szelepcsényi theorem:
 - ▶ For $f(n) \geq \log(n)$:
 $NSPACE(f(n)) = co-NSPACE(f(n))$.

Relations between time and space

- ▶ $NP \subseteq PSPACE$
 - ▶ One cannot use more than polynomial space in only polynomial time. So $NP \subseteq NPSPACE$.
 - ▶ But $NPSPACE = PSPACE$. So $NP \subseteq PSPACE$.
- ▶ $co-NP \subseteq PSPACE$
 - ▶ $co-NP$ is not included in NP because to prove a negative answer you must prove that all possible solutions are “no” and thus you must explore all possible paths. However you can re-use memory used to explore each path, thus you still need only a polynomial amount of space, and $co-NP$ is included in $PSPACE$.

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- ▶ Show that KDP is in NP.

Reducing SSP to KDP

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- ▶ A variant of subset-sum problem: given a set of n nonnegative integers $S = \{s_1, \dots, s_n\}$, does any subset of it add up to exactly T ?
- ▶ Reduce this variant of SSP to KDP.

Settling debts problem: a simple variant

- ▶ A group of friends lend each other money throughout the year. They record each transaction, and by the end of the year each of them has a final “balance”, either positive (if he lent more than borrowed), or negative or zero.
- ▶ At the end of the year they wish to settle all their debts. Money can be transferred between any pair of persons.
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- ▶ Minimize the number of transfers.
- ▶ Reduce the same variant of SSP to the above problem.