Introduction to Probability

Basic Probability Concepts

- Foundation of statistics

  because of the concept of sampling and the concept of variation or dispersion and how likely an observed difference is due to chance
- Probability statements used frequently in statistics
  - e.g., we say that we are 90% sure that an observed treatment effect in a study is real.
- Probabilities are expressed as fractions between 0.0 and 1.0
  - e.g., 0.01, 0.05, 0.10, 0.50, 0.80
  - Probability of a certain event = 1.0
  - Probability of an impossible event = 0.0

Definition of Probabilities

- If some process is repeated a large number of times, **n**, and if some resulting event with the characteristic of **A** occurs **m** times, the relative frequency of occurrence of **A**, **m/n**, will be approximately equal to the probability of **A**:

$$P(A) = \frac{no.of\ \ times\ \ A\ occurs}{no.of\ \ all\ \ possible\ \ outcome} = \frac{m}{n}$$

Permutations and Combinations

Finding P(A) simply involves counting the number of times this event occurs, but counting by hand may not feasible when the sample space is large.

Permutations:

The number of ordered sequences where repetition is not allowed, i.e. each element can be taken once only in the sequence.

$$^{n}P_{k} = \frac{n!}{(n-k)!} = n(n-1)...(n-k+1)$$

Examples:

1. Three element {1,2,3}. How many sequences of two elements from these three?
2. Four element {1, 2, 3, 4}. How many sequences of two elements from these four?

3. Find the probability that a randomly chosen four-letter sequence will not have any repeated letters.

Let I={a, b, c, …, z} be the alphabet of 26 letters. Then the sample space is

$$S=\{(\alpha, \beta, \gamma, \lambda), \alpha \in I, \beta \in I, \gamma \in I, \lambda \in I\}$$

and the event of interest is

$$E=\{(\alpha, \beta, \gamma, \lambda), \alpha \neq \beta \neq \gamma \neq \lambda\}$$

Combination

The number of unordered sequences where repetition is not allowed, i.e. each element can be taken once only in the sequence.

$$^{n}C_{k} = \frac{n!}{k!(n-k)!}$$

Examples:

1. Three element {1,2,3}. How many sequences of two elements from these three?
2. Four element {1 , 2, 3, 4}. How many sequences of two elements from these four?

3. Choose a sample of 10 from a class of 100 consisting of 60 females and 40 males. What is the probability of getting 10 females?.
4. In a party of five people, compute the probability that at least two have the same birthday (month/day), assuming a 365 day year.
5. if a box contains 75 good IC chips and 25 defective chips, and 12 chips are selected at random, find the probability that all chips are good.
6. A box with fifteen integrated circuit chips contains five defective. If a random sample three chips is drawn, what is the probability that all three are defective.

Some Properties of Probabilities

1. Probability of an event is a non-negative number
   – Given some process (or experiment) with *n* mutually exclusive outcomes (events), E_1, E_2, …, E_n, the probability of any event E_i is assigned a nonnegative number

     $P(E_i) \geq 0$

   – key concept is mutually exclusive outcomes - cannot occur simultaneously

2. Sum of the probabilities of mutually exclusive outcomes is equal to 1
   – Property of exhaustiveness
     • refers to the fact that the observer of the process must allow for all possible outcomes
   – $P(E_1) + P(E_2) + \ldots + P(E_n) = 1$
   – key concept is still mutually exclusive outcomes

3. Probability of occurrence of either of two mutually exclusive events is equal to :

   $P(A \cup B)=P(A)+P(B)$

   – If A and B are two mutually exclusive events.

   $P(A \cup B) = P(A) + P(B)-P(A \cap B)$

   – If they are not mutually exclusive.

4. For two *independent events*, A and B, occurrence of event A has no effect on probability of event B
   – $P(A \cup B) = P(A) + P(B)-P(A \cap B)$
   – $P(A/B) = P(A)$
   – $P(B/A) = P(B)$
   – $P(A \cap B) = P(A) \times P(B)$
5. Conditional probability
   – Conditional probability of B given A is given by:
   – $P(B/A) = P(A \cap B) / P(A)$
   – Probability of the occurrence of event B given that event A has already occurred.
   – Ex. given that a test for disease is positive, what is the probability that the patient has this disease?

6. Multiplicative Law
   – For any two events A and B,
   – $P(A \cap B) = P(A)\, P(B/A)$
     • Joint probability of A and B = Probability of B times Probability of A given B
7. Addition Law
   – For any two events A and B
   – $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
     • Probability of A or B = Probability of A plus Probability of B minus the joint Probability of A and B

Properties of Probabilities

Theorem 1: Complementary Events

For each $E \subset S$, $P(\overline{E}) = 1 - P(E)$

Theorem 2: The Impossible Events or the Empty Set

$P(\emptyset)=0$ where Ø is the empty set

Theorem 3: If E_1 and E_2 are subsets of S such that $E_1 \subset E_2$ then $P(E_1) \leq P(E_2)$

Theorem 4: Range of Probability

For each $E \subset S$, $0 \leq P(E) \leq 1$

Theorem 5: The Addition Law of Probability

If E_1 and E_2 are subsets of S then

$P(E_1 \cup E_2)=P(E_1)+P(E_2)-P(E_1 \cap E_2)$