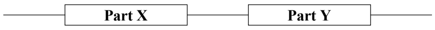
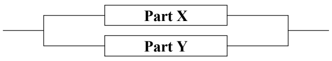
System Reliability.

A system consists of components which determine whether or not it will work. There are various types of configurations of the components in different systems.

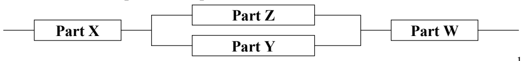
- **Series system**: this is a system in which all the components are in series and they all have to work for the system to work. If one fails, the system fails.



- **Parallel System**: this is a system that will fail only if they all fail.



- **Series-Parallel System**: this is a system where some of the components in series are replicated in parallel.



Example:

1. A system consists of 5 components in series each having a reliability of 0.67. What is the reliability of the system?.
2. A system consists of 5 components in parallel. Each component has a reliability of 0.97. The system works if at least one of them works. What is its reliability of the system?.
3. Consider a system with 5 components in series. The reliability of component 1 is 0.95, component 2 is 0.95, component 3 is 0.85, component 4 is 0.90 and component 5 is 0.95. Calculate the overall reliability of the system?.

The Reliability of a System

Reliability with series system

The problem with series system is that reliability quickly decreases as the number of components increases.

Reliability with Parallel system.

The problem with series system is that the 'law of diminishing returns' operates. The rate of increase in reliability with each additional component decreases as the number of components increases.

Bayes Theorem.

If a sample space can be partitioned into k mutually exclusive and exhaustive events:

$$A_1, A_2, \ldots, A_k$$

i.e.

$$S = A_1 \cup A_2 \cup \ldots \cup A_k$$

Then for any event E:

$$P(E) = P(A_1)P(E \backslash A_1) + \ldots + P(A_k)P(E \backslash A_k)$$

$$P(A_i \backslash E) = \frac{P(A_i)P(E \backslash A_i)}{P(E)}$$

- $P(A_i \backslash E)$ is called the **Posterior** Probability.

Example:

1. In a computer installation, 60% of programs are written in C++ and 40% in Java. 60% of the programs written in C++ compile on the first run and 80% of the Java programs compile on the first run.
   i. What is the overall proportion of programs that compile on first run?.
   ii. If a randomly selected program compile on the first run, what is the probability that it was written in C++?.
2. In a certain company, 50% of documents are written in WORD, 30% in LATEX and 20% in HTML. From past experience it is know that: 40% of the WORD documents exceed 10 pages, 20% of the LATEX documents exceed 10 pages and 20% of the HTML exceed 10 pages.
   1. What is the overall proportion of documents containing more thank 10 pages?.
   2. A document is chosen at random and found to have more than 10 pages. What is the probability that it has been written in LATEX?.

Random Variables.

Random variables are Discrete or Continuous.

- **Discrete Random Variable.**

If its values can assume isolated points on the number line.

- **Continuous Random Variable.**

It is called continuous if its values can assume all points in a particular interval.

Discrete Probability Distribution

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Binomial Distribution

Conditions:

1. An experiment consists of n repeated trials.
2. Each trial has two possible outcomes:
   a) A success with a constant probability **p** from trial to trial.
   b) A failure with probability **q=1-p**
3. Repeated trials are independent

Let **X** is a number of success in **n** trial.

PDF: $P(X=x)=\binom{n}{x}p^x q^{n-x};\ x=1,2,3,...,n$

CDF: $P(X\le x)=P(X=1)+p(X=2)+...+P(X=x)$
$=\sum_{i=1}^{x}\binom{n}{i}p^i(1-q)^{n-i}$

Expectation and Variance: **E(X)=** ***np*** and **V(X)=** ***np(1 − p).***

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Example:

1. Based on past experience, a printer in a laboratory is operating 60% of the time. Throughout a particular day, 8 visits are made and the number of times, X, that the printer which is operating is observed. What is the probability that the printer is operating:
   i. Exactly 5 times?.
   ii. At least 3 times?.
   iii. Fewer than 3 times?.
2. Suppose that each time you take a free throw shot, you have a 25% chance of making it. If you take 15 shots,
   i. What is the probability of making exactly 5 of them.
   ii. What is the probability of making fewer than 3 shots?

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Geometric Distribution

Conditions:

1. An experiment consists of repeating trials until first success.
2. Each trial has two possible outcomes:
   a) A success with probability p
   b) A failure with probability q=1-p
3. Repeated trials are independent

X is a R.V having a geometric distribution.

PDF: $P(X=x)=q^{x-1}p;\ x=1,2,3,...$

CDF: $P(X\le x)=P(X=1)+p(X=2)+...+P(X=x)$
$=p+qp+...+q^{x-1}p$
$=p(1-q^x)/(1-q)=1-q^x$

Expectation and Variance: **E(X)= 1/*p*** and **V(X)= (1 − *p*)/*p*²*.***

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Example:

1. Products produced by a machine has a 3% defective rate.
   i. What is the probability that the first defective occurs in the fifth item inspected?.
   ii. What is the probability that the first defective occurs in the first five inspections?.
2. A particularly biased coin, when tossed, will come up heads 75 % of the time. The random variable X, whose value is the number of the first toss that results in heads, is geometrically distributed with p = 0.75.
   - What is the probabilities that the first outcome of heads occurs during the first three trials?.

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Hypergeometric Distribution

A finite population of size N consists of :

1. M elements called successes.
2. N-M element called failures.

A sample of size n are selected randomly without replacement

Let **X** is a number of success.

PDF:

$$P(X=x)=\frac{\binom{M}{x}\binom{N-M}{n-x}}{\binom{N}{n}}$$

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Example: Lotto

1. 42 balls are numbered 1 – 42 and you select six numbers between for your lotto card. What is the probability that contain:
   i. Match 6?.
   ii. Match 4?.
   iii. Match 2?.

2. Suppose we randomly select 5 cards without replacement from an ordinary deck of playing cards.
   1. What is the probability of getting exactly 2 red cards (i.e., hearts or diamonds)?
   2. What is the probability of obtaining 2 or fewer hearts?

Binomial or Hypergeometric

Example: Boxes contain 20 items of which 10% are defective. Find the probability that no more than 2 defective will be obtained in a sample of size 10.

(With replacement and without replacement).

Theorem: the hypergeometric distribution converges to binomial distribution a $N \to \infty$.

Poisson Distribution

It expresses the probability of a number of events occurring in a fixed period of time if these events occur with a known average rate, and are independent of the time since the last event.

Let **X** is the number of occurrences over some interval.

PDF:

$$P(X=x)=\frac{e^{-\lambda}\lambda^{x}}{x!};\ x=1,2,3,...,\infty$$

CDF:

$$P(X\le x)=P(X=1)+p(X=2)+...+P(X=x)$$
$$=\sum_{i=1}^{x}\frac{e^{-\lambda}\lambda^{i}}{i!}$$

Expectation and Variance: **E(X)= λ** and **V(X)= λ.**

Example:

1. Suppose a bank knows that on average 60 customers arrive between 10 A.M. and 11 A.M. daily. Thus 1 customer arrives per minute. Find the probability that:
   - exactly two customers arrive in a given one-minute interval.
   - exactly **one** customer arriving in a given one minute interval.
   - **no** customers arriving in a given one minute interval.
2. Consider a computer system with Poisson job-arrival stream at an average of 2 per minute. What is the probability that:
   - Zero jobs
   - Exactly 2 jobs
   - More than 3.
3. The average rate of telephone calls received at an exchange of 8 lines is 6 per minute. Find the probability that a caller is unable to make a connection if this is defined to occur when all lines are engaged within a minute of the time of the call.

Poisson distribution as an approximation to Binomial.

When ***n*** is large and ***p*** is small, Poisson distribution is used as an approximation to the Binomial distribution, where λ=np.

$$B(x;n,p)=\binom{n}{x}p^{x}q^{n-x}\ \xrightarrow{n\to\infty}\ p(x;\lambda)=\frac{e^{-\lambda}\lambda^{x}}{x!}$$

Example:

The manufacturer of the disk drives in one of the well-known brands of microcomputer expects 2% of the drives to malfunction during the warranty period. Calculate the probability that in a sample of 100 disk drives, that not more than three will malfunction.

P.D.F./C.D.F.

- If X is a R.V. with a finite countable set of possible outcomes, $\{x_1, x_2, \ldots\}$, then the ***discrete probability distribution*** of X

$$f(x) \;\; or \;\; p_x(x_i) = \begin{cases} P\{X = x_i\} & if \; x = x_i, \quad i = 1,2,.... \\ 0 & if \; x \neq x_i \end{cases}$$

and ***D.F. or C.D.F.***

$$P\{X \leq x_i\} = F(x_i) = \sum_{i \leq x_j} P\{X = x_i\}$$

- While, similarly, for X a R.V. taking any value **along an interval** of the **real** number line

$$F(x) = P\{X \leq x\} = \int_{-\infty}^{x} f(u)du$$

So if first derivative $F'(x)$ exists, then

$$F'(x) = dF(x)/dx = f(x)$$

$f(x) = F'(x)$ is the ***continuous pdf***, with $\int_{-\infty}^{\infty} f(x)dx = 1$

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EXPECTATION/VARIANCE

- Clearly,

$$E(X) = \begin{cases} \sum_{i \in S} x_i f(x_i) & discrete \\ \int_{-\infty}^{\infty} x f(x) dx & continuous \end{cases}$$

- and

$$Var(X) = \begin{cases} \sum_{x \in S} [x_i - E(X)]^2 f(x_i) & discrete \\ \int_{-\infty}^{\infty} [x - E(X)]^2 f(x) dx & continuous \end{cases}$$

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PROPERTIES - Expectation/Variance.

1. For R.V. X with p.d.f., the expected values for any function *g(x) is given as*

$$E(g(x)) = \begin{cases} \sum_{x} g(x) f(x) & discrete \\ \int_{-\infty}^{\infty} g(x) f(x) dx & continuous \end{cases}$$

2. $E(aX + b) = aE(X) + b$, where a and b are constant.
3. $E(X + Y) = E(X) + E(Y)$, where X and Y are R.V's.
4. $V(aX + b) = a^2 V(X)$.
5. $V(X + Y) = V(X) + V(Y) + 2COV(X, Y)$.
   - V(X+Y)=V(X)+V(Y), if they are independent.

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Examples:

1. A quarter of the source programs submitted by a certain programmer compile successfully. Each day the programmer writes five programs. The compiling probabilities are:

No. that compiles	0	1	2	3	4	5
Probability	0.237	0.396	0.264	0.088	0.014	0.001

Calculate:

- The expected number of programs that will compile per day
- Variance

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Examples:

2. The average salary of employees in a company is 30,200 euro. After negotiations with the management, it was agreed that they would get a rise of 200 euro in addition to 5 percent increase on their old salaries. What is the new average salary?.

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