

# CA313 Algorithms and Complexity

## Class Test 3

Attempt **all** questions. All questions carry equal marks. You have **45 mins.** to attempt these questions.

Name:

Student Number:

Q 1.

(i) Define *time complexity*.

(ii) Define *space complexity*.

(iii) Polynomial solutions are usually considered efficient, while exponential solutions are considered inefficient. Give examples which show that this is not completely true.

(iv) Give the growth rates for:

(a)  $n^2$  compared to  $2^n$ , for values of  $n$  from 1–8;

(b)  $n^3$  compared to  $2^n$ , for values of  $n$  from 1–11;

Comment on the practical feasibility of algorithms with complexity  $n^2$ ,  $2^n$  and  $n^3$  with respect to inputs of these lengths  $n$ .

Q 2.

(i) Name the following complexities (e.g.  $O(n^2)$  is ‘quadratic’), and for each of them, say if they are usually considered efficient (e.g. quadratic complexities *are* considered efficient).

- $O(3)$ :
- $O(n)$ :
- $O(\exp(n))$ :
- $O(n^k)$  ( $k$  is a natural number):
- $O(\ln(n))$ :
- $O(n^3)$ :
- $O(2^n)$ :

(ii) Write the following functions using the  $O$  notation (e.g.  $2n = O(n)$ ).

- $1000n^2$ :
- $5\ln(n) + 10$ :
- $\exp(n) + 3n^2$ :
- $2n^3 + 500n^2$ :
- $3n^2 \ln(n)$ :
- $2^{(n/2)} + \ln(n)$ :

Q 3.

(i) What is a binary decision problem?

(ii) Give an example of a binary decision problem.

(iii) Define the class  $P$ .

(iv) Define the class  $NP$ .

Q 4.

We want to print the smallest integer in a matrix of integers  $M$  which has  $r$  rows and  $c$  columns. We use the following algorithm:

```
1  int min=0;
2  for (i=0; i<r; i++){
3      for (j=0; j<c; j++){
4          if ( M[i][j] < min ){
5              min = M[i][j];
6          }
7      }
8  }
9  print min;
```

(i) In the worst case, what is the number of elemental instructions  $T$  needed to run (once) the following parts of the algorithm:

- lines 4 to 6
- the inner loop (lines 3 to 7)
- the outer loop (lines 2 to 8)
- the whole algorithm (lines 1 to 9)

(ii) What is the time complexity of this algorithm (using the  $O$  notation, e.g.  $2n = O(n)$ ) ?

(iii) Consider the following problem: “Is a given integer  $n$  the minimum of the matrix  $M$  ?”. To solve it we can use the above algorithm to find the minimum and check whether it is equal to  $n$ . Is this decision problem in class  $P$ ? In class  $NP$ ? Justify your answer.