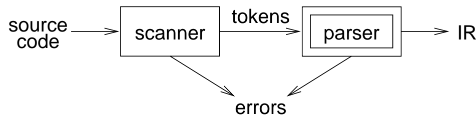
# The role of the parser




Parser:

- performs context-free syntax analysis
- guides context-sensitive analysis
- constructs an intermediate representation
- produces meaningful error messages
- attempts error correction



# Syntax analysis

*Context free syntax* is specified with a context free grammar.

Formally, a `CFG` $G$ is a 4-tuple $(V_t, V_n, S, P)$, where:

$V_t$ is the set of terminal symbols in the grammar.
For our purposes, $V_t$ is the set of tokens returned by the scanner.

$V_n$ are the *nonterminals*, a set of syntactic variables that denote sets of (sub)strings occurring in the language.
These are used to impose a structure on the grammar.

$S$ is a distinguished nonterminal ($S \in V_n$) denoting the entire set of strings in $L(G)$.
This is sometimes called a *goal symbol*.

$P$ is a finite set of *productions* specifying how terminals and non-terminals can be combined to form strings in the language.
Each production must have a single non-terminal on its left hand side.

The set $V = V_t \cup V_n$ is called the *vocabulary* of $G$





# Notation and terminology

- $a, b, c, ... \in V_t$
- $A, B, C, ... \in V_n$
- $U, V, W, ... \in V$
- $\alpha, \beta, \gamma, ... \in V^*$
- $u, v, w, ... \in V_t^*$

If $A \rightarrow \gamma$ then $\alpha A \beta \Rightarrow \alpha \gamma \beta$ is a *single-step derivation* using $A \rightarrow \gamma$

Similarly, $\Rightarrow^*$ and $\Rightarrow^+$ denote derivations of $\geq 0$ and $\geq 1$ steps

If $S \Rightarrow^* \beta$ then $\beta$ is said to be a *sentential form* of $G$

$L(G) = \{w \in V_t^* | S \Rightarrow^+ w\}, w \in L(G)$ is called a *sentence* of $G$

Note, $L(G) = \{\beta \in V^* | S \Rightarrow^* \beta\} \cap V_t^*$





# Syntax analysis

Grammars are often written in Backus-Naur form (BNF).

Example:

| | | | |
|---|---|---|---|
| 1 | `<goal>` | ::= | `<expr>` |
| 2 | `<expr>` | ::= | `<expr><op><expr>` |
| 3 | | \| | `num` |
| 4 | | \| | `id` |
| 5 | `<op>` | ::= | $+$ |
| 6 | `<op>` | ::= | $-$ |
| 7 | `<op>` | ::= | $*$ |
| 8 | `<op>` | ::= | $/$ |

This describes simple expressions over numbers and identifiers.

In a BNF for a grammar, we represent

1. non-terminals with angle brackets or capital letters
2. terminals with `typewriter` font or <u>underline</u>
3. productions as in the example

## Scanning vs. parsing

*Where do we draw the line?*

$term$ ::= [a–zA–Z]([a–zA–Z] | [0–9])*
  | 0 | [1–9][0–9]*
$op$ ::= $+ \mid - \mid * \mid /$
$expr$ ::= $(term\ op)^* term$

Regular expressions are used to classify:

- identifiers, numbers, keywords
- REs are more concise and easier to understand for tokens than a grammar
- more efficient scanners (DFAs) can be built from REs than from arbitrary grammars

Context free grammars are used to count:

- brackets: `()`,`begin...end`,`if...then...else`
- imparting structure: expressions

Syntactic analysis is complicated enough: grammar for C has around 200 productions. Factoring out lexical analysis as a separate phase makes the compiler more manageable.



## Derivations

We can view the productions of a CFG as rewriting rules.

Using our example CFG:

<goal> ⇒ <expr>
⇒ <expr><op><expr>
⇒ <expr><op><expr><op><expr>
⇒ <id,x><op><expr><op><expr>
⇒ <id,x> + <expr><op><expr>
⇒ <id,x> + <num,2><op><expr>
⇒ <id,x> + <num,2>*<expr>
⇒ <id,x> + <num,2>*<id,y>

We have derived the sentence `x` + `2*y`.

We denote this <goal> $\Rightarrow^*$ `id` + `num*id`.

Such a sequence of rewrites is a *derivation* or a *parse*.

The process of discovering a derivation is called *parsing*.





## Derivations

*At each step, we chose a non-terminal to replace.*

*This choice can lead to different derivations.*

Two are of particular interest:

*leftmost derivation*

the leftmost non-terminal is replaced at each step

*rightmost derivation*

the rightmost non-terminal is replaced at each step

*The previous example was a leftmost derivation.*





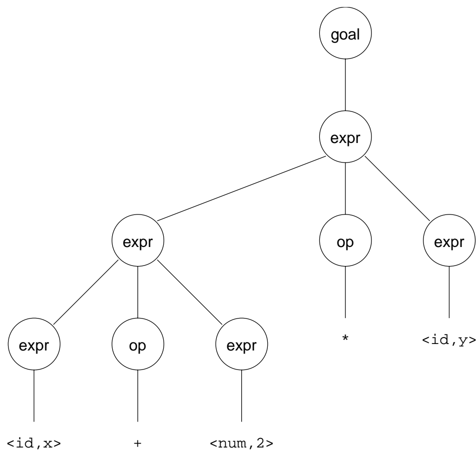
## Rightmost derivation

For the string `x` + `2*y`:

<goal> ⇒ <expr>
⇒ <expr><op><expr>
⇒ <expr><op><id,y>
⇒ <expr>*<id,y>
⇒ <expr><op><expr>*<id,y>
⇒ <expr><op><num,2>*<id,y>
⇒ <expr> + <num,2>*<id,y>
⇒ <id,x> + <num,2>*<id,y>

Again, <goal> $\Rightarrow^*$ `id` + `num*id`.

# Precedence




*Treewalk evaluation computes* (x + 2) * y
— the "wrong" answer!

Should be x + (2 * y)



# Precedence

*These two derivations point out a problem with the grammar.*
*It has no notion of precedence, or implied order of evaluation.*
To add precedence takes additional machinery:

| | | | |
|---|---|---|---|
| 1 | <goal> | ::= | <expr> |
| 2 | <expr> | ::= | <expr> + <term> |
| 3 | | \| | <expr> - <term> |
| 4 | | \| | <term> |
| 5 | <term> | ::= | <term> * <factor> |
| 6 | | \| | <term> / <factor> |
| 7 | | \| | <factor> |
| 8 | <factor> | ::= | num |
| 9 | | \| | id |

This grammar enforces a precedence on the derivation:

- terms *must* be derived from expressions
- forces the "correct" tree





# Precedence

Now, for the string x + 2 * y:

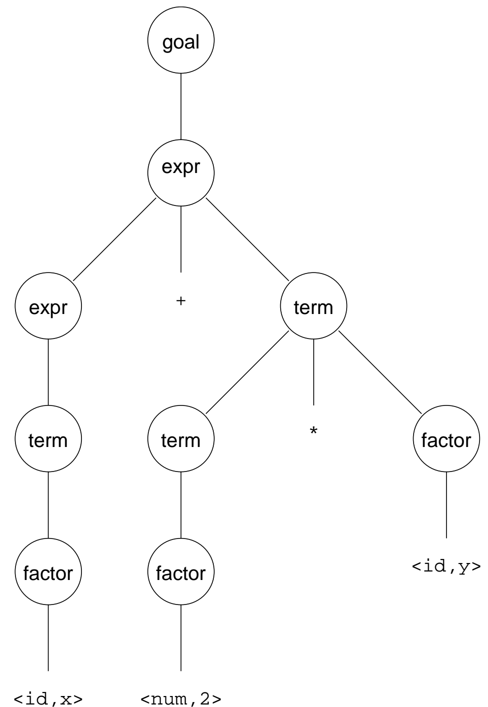
| | | |
|---|---|---|
| <goal> | ⇒ | <expr> |
| | ⇒ | <expr> + <term> |
| | ⇒ | <expr> + <term> * <factor> |
| | ⇒ | <expr> + <term> * <id,y> |
| | ⇒ | <expr> + <factor> * <id,y> |
| | ⇒ | <expr> + <num,2> * <id,y> |
| | ⇒ | <term> + <num,2> * <id,y> |
| | ⇒ | <factor> + <num,2> * <id,y> |
| | ⇒ | <id,x> + <num,2> * <id,y> |

Again, <goal>$\Rightarrow^*$ id + num * id, but this time, we build the desired tree.





# Precedence




*Treewalk evaluation computes* x + (2 * y)

## Ambiguity

If a grammar has more than one derivation for a single sentential form, then it is *ambiguous*

Example:

```
<stmt>  ::=  if <expr> then <stmt>
          |  if <expr> then <stmt> else <stmt>
          |  other stmts
```

Consider deriving the sentential form:

$$\texttt{if } E_1 \texttt{ then if } E_2 \texttt{ then } S_1 \texttt{ else } S_2$$

It has two derivations.

This ambiguity is purely grammatical.

It is a *context free* ambiguity.



## Ambiguity

May be able to eliminate ambiguities by rearranging the grammar:

```
<stmt>      ::= <matched>
              | <unmatched>
<matched>   ::= if <expr> then <matched> else <matched>
              | other stmts
<unmatched> ::= if <expr> then <stmt>
              | if <expr> then <matched> else <unmatched>
```

This generates the same language as the ambiguous grammar, but applies the common sense rule:

*match each* `else` *with the closest unmatched* `then`

This is most likely the language designer's intent.





## Ambiguity

*Ambiguity* is often due to confusion in the context free specification.

Context sensitive confusions can arise from *overloading*.

Example:

```
a = f(17)
```

In many Algol-like languages, `f` could be a function or a subscripted variable.

Disambiguating this statement requires context:

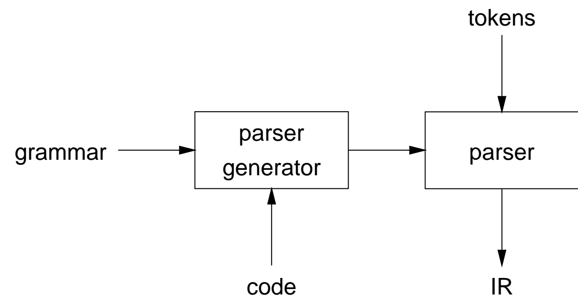
- need *values* of declarations
- not *context free*
- really an issue of *type*

*Rather than complicate parsing, we will handle this separately.*





## Parsing: the big picture




*Our goal is a flexible parser generator system*

## Top-down versus bottom-up

*Top-down parsers*

- start at the root of derivation tree and fill in
- picks a production and tries to match the input
- may require backtracking
- some grammars are backtrack-free (*predictive*)

*Bottom-up parsers*

- start at the leaves and fill in
- start in a state valid for legal first tokens
- as input is consumed, change state to encode possibilities (*recognize valid prefixes*)
- use a stack to store both state and sentential forms



## Top-down parsing

*A top-down parser starts with the root of the parse tree, labelled with the start or goal symbol of the grammar.*

To build a parse, it repeats the following steps until the fringe of the parse tree matches the input string

1. At a node labelled $A$, select a production $A \rightarrow \alpha$ and construct the appropriate child for each symbol of $\alpha$
2. When a terminal is added to the fringe that doesn't match the input string, backtrack
3. Find the next node to be expanded (must have a label in $V_n$)

The key is selecting the right production in step 1

$\Rightarrow$ should be guided by the input string





## Simple expression grammar

Recall our grammar for simple expressions:

| | | | |
|---|---|---|---|
| 1 | <goal> | ::= | <expr> |
| 2 | <expr> | ::= | <expr> + <term> |
| 3 | | \| | <expr> - <term> |
| 4 | | \| | <term> |
| 5 | <term> | ::= | <term> * <factor> |
| 6 | | \| | <term> / <factor> |
| 7 | | \| | <factor> |
| 8 | <factor> | ::= | num |
| 9 | | \| | id |

Consider the input string `x - 2 * y`





## Example

| Prod'n | Sentential form | Input |
|---|---|---|
| – | <goal> | $\uparrow x - 2 * y$ |
| 1 | <expr> | $\uparrow x - 2 * y$ |
| 2 | <expr> + <term> | $\uparrow x - 2 * y$ |
| 4 | <term> + <term> | $\uparrow x - 2 * y$ |
| 7 | <factor> + <term> | $\uparrow x - 2 * y$ |
| 9 | id + <term> | $\uparrow x - 2 * y$ |
| – | id + <term> | $x \uparrow - 2 * y$ |
| – | <expr> | $\uparrow x - 2 * y$ |
| 3 | <expr> - <term> | $\uparrow x - 2 * y$ |
| 4 | <term> - <term> | $\uparrow x - 2 * y$ |
| 7 | <factor> - <term> | $\uparrow x - 2 * y$ |
| 9 | id - <term> | $\uparrow x - 2 * y$ |
| – | id - <term> | $x \uparrow - 2 * y$ |
| – | id - <term> | $x - \uparrow 2 * y$ |
| 7 | id - <factor> | $x - \uparrow 2 * y$ |
| 8 | id - num | $x - \uparrow 2 * y$ |
| – | id - num | $x - 2 \uparrow * y$ |
| – | id - <term> | $x - \uparrow 2 * y$ |
| 5 | id - <term> * <factor> | $x - \uparrow 2 * y$ |
| 7 | id - <factor> * <factor> | $x - \uparrow 2 * y$ |
| 8 | id - num * <factor> | $x - \uparrow 2 * y$ |
| – | id - num * <factor> | $x - 2 \uparrow * y$ |
| – | id - num * <factor> | $x - 2 * \uparrow y$ |
| 9 | id - num * id | $x - 2 * \uparrow y$ |
| – | id - num * id | $x - 2 * y \uparrow$ |

## Example

Another possible parse for `x - 2 * y`

| Prod'n | Sentential form | Input |
|---|---|---|
| – | <goal> | $\uparrow x - 2 * y$ |
| 1 | <expr> | $\uparrow x - 2 * y$ |
| 2 | <expr> + <term> | $\uparrow x - 2 * y$ |
| 2 | <expr> + <term> + <term> | $\uparrow x - 2 * y$ |
| 2 | <expr> + <term> + ... | $\uparrow x - 2 * y$ |
| 2 | <expr> + <term> + ... | $\uparrow x - 2 * y$ |
| 2 | ... | $\uparrow x - 2 * y$ |

If the parser makes wrong choices, expansion doesn't terminate.

This isn't a good property for a parser to have.

(Parsers should terminate!)



## Left-recursion

*Top-down parsers cannot handle left-recursion in a grammar*

Formally, a grammar is *left-recursive* if

$\exists A \in V_n$ such that $A \Rightarrow^+ A\alpha$ for some string $\alpha$

*Our simple expression grammar is left-recursive*





## Eliminating left-recursion

*To remove left-recursion, we can transform the grammar*

Consider the grammar fragment:

$$\begin{array}{lcl} A & ::= & A\alpha \\ & | & \beta \end{array}$$

where $\alpha$ and $\beta$ do not start with $A$

We can rewrite this as:

$$\begin{array}{lcl} A & ::= & \beta A' \\ A' & ::= & \alpha A' \\ & | & \epsilon \end{array}$$

where $A'$ is a new non-terminal

*This fragment contains no left-recursion*





## Example

Our expression grammar contains two cases of left-recursion

```
<expr>  ::= <expr> + <term>
          | <expr> - <term>
          | <term>
<term>  ::= <term> * <factor>
          | <term> / <factor>
          | <factor>
```

Applying the transformation gives

```
<expr>  ::= <term> <expr'>
<expr'> ::= + <term> <expr'>
          | - <term> <expr'>
          | ε
<term>  ::= <factor> <term'>
<term'> ::= * <factor> <term'>
          | / <factor> <term'>
          | ε
```

With this grammar, a top-down parser will

- terminate
- backtrack on some inputs

## Example

This cleaner grammar defines the same language

```
1  <goal>    ::=  <expr>
2  <expr>    ::=  <term> + <expr>
3              |  <term> - <expr>
4              |  <term>
5  <term>    ::=  <factor> * <term>
6              |  <factor> / <term>
7              |  <factor>
8  <factor>  ::=  num
9              |  id
```

It is

- right-recursive
- free of $\epsilon$ productions

*Unfortunately, it generates different associativity*

*Same syntax, different meaning*



## Example

The expression grammar:

```
1   <goal>    ::=  <expr>
2   <expr>    ::=  <term> <expr'>
3   <expr'>   ::=  + <term> <expr'>
4               |  - <term> <expr'>
5               |  ε
6   <term>    ::=  <factor> <term'>
7   <term'>   ::=  * <factor> <term'>
8               |  / <factor> <term'>
9               |  ε
10  <factor>  ::=  num
11              |  id
```

Recall, we factored out left-recursion





## How much lookahead is needed?

*We saw that top-down parsers may need to backtrack when they select the wrong production*

Do we need arbitrary lookahead to parse CFGs?

- in general, yes
- use the Earley or Cocke-Younger, Kasami algorithms

Fortunately

- large subclasses of CFGs can be parsed with limited lookahead
- most programming language constructs can be expressed in a grammar that falls in these subclasses

Among the interesting subclasses are:

**LL(1):** **L**eft to right scan, **L**eft-most derivation, **1**-token lookahead

**LR(1):** **L**eft to right scan, **R**ight-most derivation, **1**-token lookahead





## Predictive parsing

*Basic idea:*

For any two productions $A \rightarrow \alpha | \beta$, we would like a distinct way of choosing the correct production to expand.

For some RHS $\alpha \in G$, define $\texttt{FIRST}(\alpha)$ as the set of tokens that appear first in some string derived from $\alpha$

That is, for some $w \in V_t^*$ , $w \in \texttt{FIRST}(\alpha)$ iff. $\alpha \Rightarrow^* w\gamma$.

*Key property:*

Whenever two productions $A \rightarrow \alpha$ and $A \rightarrow \beta$ both appear in the grammar, we would like

$$\texttt{FIRST}(\alpha) \cap \texttt{FIRST}(\beta) = \emptyset$$

This would allow the parser to make a correct choice with a lookahead of only one symbol!

*The example grammar has this property!*

## Left factoring

*What if a grammar does not have this property?*

Sometimes, we can transform a grammar to have this property.

For each non-terminal $A$ find the longest prefix $\alpha$ common to two or more of its alternatives.

if $\alpha \neq \epsilon$ then replace all of the $A$ productions

$$A \rightarrow \alpha\beta_1 | \alpha\beta_2 | ... | \alpha\beta_n$$

with

$$A \rightarrow \alpha A'$$
$$A' \rightarrow \beta_1 | \beta_2 | ... | \beta_n$$

where $A'$ is a new non-terminal.

Repeat until no two alternatives for a single non-terminal have a common prefix.



## Example

Consider a *right-recursive* version of the expression grammar:

| | | | |
|---|---|---|---|
| 1 | <goal> | ::= | <expr> |
| 2 | <expr> | ::= | <term> + <expr> |
| 3 | | \| | <term> - <expr> |
| 4 | | \| | <term> |
| 5 | <term> | ::= | <factor> * <term> |
| 6 | | \| | <factor> / <term> |
| 7 | | \| | <factor> |
| 8 | <factor> | ::= | num |
| 9 | | \| | id |

To choose between productions 2, 3, & 4, the parser must see past the `num` or `id` and look at the +, -, * or /.

$$\texttt{FIRST}(2) \cap \texttt{FIRST}(3) \cap \texttt{FIRST}(4) \neq \emptyset$$

This grammar *fails* the test.

Note: *This grammar is right-associative.*





## Example

There are two nonterminals that must be left factored:

| | | |
|---|---|---|
| <expr> | ::= | <term> + <expr> |
| | \| | <term> - <expr> |
| | \| | <term> |
| <term> | ::= | <factor> * <term> |
| | \| | <factor> / <term> |
| | \| | <factor> |

Applying the transformation gives us:

| | | |
|---|---|---|
| <expr> | ::= | <term> <expr′> |
| <expr′> | ::= | + <expr> |
| | \| | - <expr> |
| | \| | $\epsilon$ |
| <term> | ::= | <factor> <term′> |
| <term′> | ::= | * <term> |
| | \| | / <term> |
| | \| | $\epsilon$ |





## Example

Substituting back into the grammar yields

| | | | |
|---|---|---|---|
| 1 | <goal> | ::= | <expr> |
| 2 | <expr> | ::= | <term> <expr′> |
| 3 | <expr′> | ::= | + <expr> |
| 4 | | \| | - <expr> |
| 5 | | \| | $\epsilon$ |
| 6 | <term> | ::= | <factor> <term′> |
| 7 | <term′> | ::= | * <term> |
| 8 | | \| | / <term> |
| 9 | | \| | $\epsilon$ |
| 10 | <factor> | ::= | num |
| 11 | | \| | id |

Now, selection requires only a single token lookahead.

Note: *This grammar is still right-associative.*

# Example

| Prod'n | Sentential form | Input |
|---|---|---|
| – | <goal> | $\uparrow x \; - \; 2 \; * \; y$ |
| 1 | <expr> | $\uparrow x \; - \; 2 \; * \; y$ |
| 2 | <term> <expr'> | $\uparrow x \; - \; 2 \; * \; y$ |
| 6 | <factor> <term'> <expr'> | $\uparrow x \; - \; 2 \; * \; y$ |
| 11 | id <term'> <expr'> | $\uparrow x \; - \; 2 \; * \; y$ |
| – | id <term'> <expr'> | $x\uparrow - \; 2 \; * \; y$ |
| 9 | id <expr'> | $x\uparrow - \; 2 \; * \; y$ |
| 4 | id - <expr> | $x\uparrow - \; 2 \; * \; y$ |
| – | id - <expr> | $x \; -\uparrow 2 \; * \; y$ |
| 2 | id - <term> <expr'> | $x \; -\uparrow 2 \; * \; y$ |
| 6 | id - <factor> <term'> <expr'> | $x \; -\uparrow 2 \; * \; y$ |
| 10 | id - num <term'> <expr'> | $x \; -\uparrow 2 \; * \; y$ |
| – | id - num <term'> <expr'> | $x \; - \; 2\uparrow * \; y$ |
| 7 | id - num * <term> <expr'> | $x \; - \; 2\uparrow * \; y$ |
| – | id - num * <term> <expr'> | $x \; - \; 2 \; *\uparrow y$ |
| 6 | id - num * <factor> <term'> <expr'> | $x \; - \; 2 \; *\uparrow y$ |
| 11 | id - num * id <term'> <expr'> | $x \; - \; 2 \; *\uparrow y$ |
| – | id - num * id <term'> <expr'> | $x \; - \; 2 \; * \; y\uparrow$ |
| 9 | id - num * id <expr'> | $x \; - \; 2 \; * \; y\uparrow$ |
| 5 | id - num * id | $x \; - \; 2 \; * \; y\uparrow$ |

The next symbol determined each choice correctly.



# Back to left-recursion elimination

Given a left-factored CFG, to eliminate left-recursion:

if $\exists A \rightarrow A\alpha$ then replace all of the $A$ productions

$$A \rightarrow A\alpha|\beta|\ldots|\gamma$$

with

$$A \rightarrow NA'$$
$$N \rightarrow \beta|\ldots|\gamma$$
$$A' \rightarrow \alpha A'|\epsilon$$

where $N$ and $A'$ are new productions.

Repeat until there are no left-recursive productions.





# Generality

Question:

> By *left factoring* and *eliminating left-recursion*, can we transform an arbitrary context free grammar to a form where it can be predictively parsed with a single token lookahead?

Answer:

> Given a context free grammar that doesn't meet our conditions, it is undecidable whether an equivalent grammar exists that does meet our conditions.

Many *context free languages* do not have such a grammar:

$$\{a^n 0 b^n | n \geq 1\} \bigcup \{a^n 1 b^{2n} | n \geq 1\}$$

Must look past an arbitrary number of $a$'s to discover the 0 or the 1 and so determine the derivation.





# Recursive descent parsing

Now, we can produce a simple recursive descent parser from the (right-associative) grammar.

```
void goal()
{
   token = next_token();
   expr();
   if (token != EOF) then
      error();
}

void expr()
{
   term();
   expr_prime();
}

void expr_prime()
{
   if ((token == PLUS) || (token == MINUS)) then
   {
      token = next_token();
      expr();
   }
}
```

## Recursive descent parsing

```
void term()
{
   factor();
   term_prime();
}

void term_prime()
{
   if ((token == MULT) || (token == DIV)) then
   {
      token = next_token();
      term();
   }
}

void factor()
{
   if ((token == NUM) || (token == ID)) then
      token = next_token();
   else
      error();
}
```



## Building the tree

*One of the key jobs of the parser is to build an intermediate representation of the source code.*

To build an abstract syntax tree, we can simply insert code at the appropriate points:

- `factor()` can stack nodes `id`, `num`
- `term_prime()` can stack nodes *, /
- `term()` can pop 3, build and push subtree
- `expr_prime()` can stack nodes +, -
- `expr()` can pop 3, build and push subtree
- `goal()` can pop and return tree





## Non-recursive predictive parsing

Observation:

> *Our recursive descent parser encodes state information in its run-time stack, or call stack.*

Using recursive procedure calls to implement a stack abstraction may not be particularly efficient.

This suggests other implementation methods:

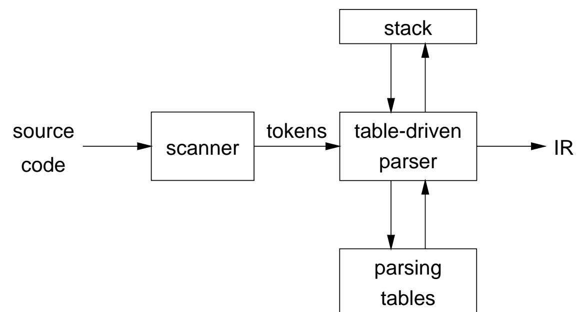
- explicit stack, hand-coded parser
- stack-based, table-driven parser





## Non-recursive predictive parsing

Now, a predictive parser looks like:

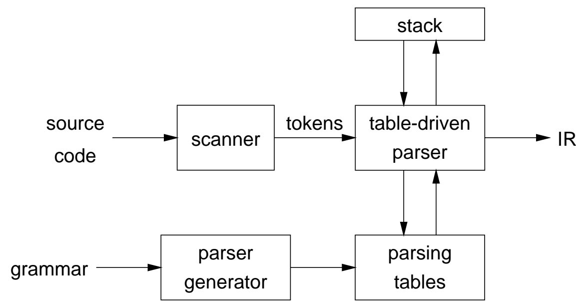



Rather than writing code, we build tables.

Building tables can be automated!

## Table-driven parsers

A parser generator system often looks like:




This is true for both top-down and bottom-up parsers:

- LL(1)
- LR(1)



## Non-recursive predictive parsing

*Input*: a string $w$ and a parsing table $M$ for $G$

```
tos = 0
Stack[tos] = EOF
Stack[++tos] =  Start Symbol
token = next_token()
repeat
   X = Stack[tos]
   if X is a terminal or EOF then
      if X == token then
         pop X
         token = next_token()
      else error()
   else /* X is a non-terminal */
      if M[X,token] == X → Y1Y2...Yk then
         pop X
         push Yk, Yk-1, ..., Y1
      else error()
until X == EOF
```





## Non-recursive predictive parsing

*What we need now is a parsing table M.*
The expression grammar:

| | | | |
|---|---|---|---|
| 1 | <goal> | ::= | <expr> |
| 2 | <expr> | ::= | <term> <expr′> |
| 3 | <expr′> | ::= | + <expr> |
| 4 | | \| | - <expr> |
| 5 | | \| | $\epsilon$ |
| 6 | <term> | ::= | <factor> <term′> |
| 7 | <term′> | ::= | * <term> |
| 8 | | \| | / <term> |
| 9 | | \| | $\epsilon$ |
| 10 | <factor> | ::= | num |
| 11 | | \| | id |

Its parse table:

| | id | num | + | - | * | / | $ |
|---|---|---|---|---|---|---|---|
| <goal> | 1 | 1 | – | – | – | – | – |
| <expr> | 2 | 2 | – | – | – | – | – |
| <expr′> | – | – | 3 | 4 | – | – | 5 |
| <term> | 6 | 6 | – | – | – | – | – |
| <term′> | – | – | 9 | 9 | 7 | 8 | 9 |
| <factor> | 11 | 10 | – | – | – | – | – |





## FIRST

For a string of grammar symbols $\alpha$, we define FIRST($\alpha$) as:

- the set of terminal symbols that begin strings derived from $\alpha$: $\{a \in V_t | \alpha \Rightarrow^* a\beta\}$
- If $\alpha \Rightarrow^* \epsilon$ then $\epsilon \in$ FIRST($\alpha$)

FIRST($\alpha$) contains the set of tokens valid in the initial position in $\alpha$
To build FIRST($X$):

1. If $X \in V_t$ then FIRST($X$) is $\{X\}$
2. If $X \rightarrow \epsilon$ then add $\epsilon$ to FIRST($X$).
3. If $X \rightarrow Y_1Y_2...Y_k$:
   (a) Put FIRST($Y_1$)–$\{\epsilon\}$ in FIRST($X$)
   (b) $\forall i : 1 < i \leq k$, if $\epsilon \in$FIRST($Y_1$)∩...∩FIRST($Y_{i-1}$)
   (i.e., $Y_1...Y_{i-1} \Rightarrow^* \epsilon$)
   then put FIRST($Y_i$) - $\{\epsilon\}$ in FIRST($X$)
   (c) If $\epsilon \in$ FIRST($Y_1$)∩...∩FIRST($Y_k$) then put $\epsilon$ in FIRST($X$)
   Repeat until no more additions can be made.

# FOLLOW

For a non-terminal $B$, we define FOLLOW($B$) as:

the set of terminals that can appear immediately to the right of $B$ in some sentential form

Thus, a non-terminal's FOLLOW set specifies the tokens that can legally appear after it.

A terminal symbol has no FOLLOW set.

To build FOLLOW($B$):

1. Put \$ in FOLLOW(<goal>)

2. If $A \rightarrow \alpha B \beta$:

   (a) Put FIRST($\beta$) - $\{\epsilon\}$ in FOLLOW($B$)
   (b) If $\beta = \epsilon$ (i.e., $A \rightarrow \alpha B$) or $\epsilon \in$ FIRST($\beta$) (i.e., $\beta \Rightarrow^* \epsilon$)
   then put FOLLOW($A$) in FOLLOW($B$)

   Repeat until no more additions can be made



# LOOKAHEAD

For a production rule $A \rightarrow \alpha$, we define LOOKAHEAD($A \rightarrow \alpha$) as:

the set of terminals which can appear next in the input when recognising production rule $A \rightarrow \alpha$

Thus, a production rule's LOOKAHEAD set specifies the tokens which should appear next in the input before the production rule is applied.

To build LOOKAHEAD($A \rightarrow \alpha$):

1. Put FIRST($\alpha$) - $\{\epsilon\}$ in LOOKAHEAD($A \rightarrow \alpha$)

2. If $\epsilon \in$ FIRST($\alpha$)
   then put FOLLOW($A$) in LOOKAHEAD($A \rightarrow \alpha$)

A grammar $G$ is LL(1) iff. for each set of productions $A \rightarrow \alpha_1|\alpha_2|...|\alpha_n$:

LOOKAHEAD($A \rightarrow \alpha_1$),LOOKAHEAD($A \rightarrow \alpha_2$),..., LOOKAHEAD($A \rightarrow \alpha_n$) are all pairwise disjoint





# LL(1) grammars

Provable facts about LL(1) grammars:

1. No left-recursive grammar is LL(1)

2. No ambiguous grammar is LL(1)

3. Some languages have no LL(1) grammar

4. A $\epsilon$–free grammar where each alternative expansion for $A$ begins with a distinct terminal is a *simple* LL(1) grammar.

Example

$S \rightarrow aS|a$

is not LL(1) because LOOKAHEAD($S \rightarrow aS$) $=$ LOOKAHEAD($S \rightarrow a$) $= \{a\}$

$S \rightarrow aS'$
$S' \rightarrow aS'|\epsilon$

accepts the same language and is LL(1)





# LL(1) parse table construction

*Input*: Grammar $G$

*Output*: Parsing table $M$

*Method*:

1. $\forall$ productions $A \rightarrow \alpha$:
   $\forall a \in$ LOOKAHEAD($A \rightarrow \alpha$), add $A \rightarrow \alpha$ to $M[A, a]$

2. Set each undefined entry of $M$ to `error`

If $\exists M[A, a]$ with multiple entries then grammar is not LL(1).

## Example

The expression grammar:

$$
\begin{array}{l|l}
S \to E & T \to FT' \\
E \to TE' & T' \to *T \mid /T \mid \epsilon \\
E' \to +E \mid -E \mid \epsilon & F \to \texttt{id} \mid \texttt{num}
\end{array}
$$

| | FIRST | FOLLOW |
|---|---|---|
| $S$ | {num,id} | {\$} |
| $E$ | {num,id} | {\$} |
| $E'$ | $\{\epsilon, +, -\}$ | {\$} |
| $T$ | {num,id} | {+,-,\$} |
| $T'$ | $\{\epsilon, *, /\}$ | {+,-,\$} |
| $F$ | {num,id} | {+,-,*,/,\$} |
| id | {id} | – |
| num | {num} | – |
| * | {*} | – |
| / | {/} | – |
| + | {+} | – |
| - | {-} | – |

| | LOOKAHEAD |
|---|---|
| $S \to E$ | {num,id} |
| $E \to TE'$ | {num,id} |
| $E' \to +E$ | {+} |
| $E' \to -E$ | {-} |
| $E' \to \epsilon$ | {\$} |
| $T \to FT'$ | {num,id} |
| $T' \to *T$ | {*} |
| $T' \to /T$ | {/} |
| $T' \to \epsilon$ | {+,-,\$} |
| $F \to$ id | {id} |
| $F \to$ num | {num} |

| | id | num | + | - | * | / | \$ |
|---|---|---|---|---|---|---|---|
| $S$ | $S \to E$ | $S \to E$ | – | – | – | – | – |
| $E$ | $E \to TE'$ | $E \to TE'$ | – | – | – | – | – |
| $E'$ | – | – | $E' \to +E$ | $E' \to -E$ | – | – | $E' \to \epsilon$ |
| $T$ | $T \to FT'$ | $T \to FT'$ | – | – | – | – | – |
| $T'$ | – | – | $T' \to \epsilon$ | $T' \to \epsilon$ | $T' \to *T$ | $T' \to /T$ | $T' \to \epsilon$ |
| $F$ | $F \to$ id | $F \to$ num | – | – | – | – | – |



## Building the tree

Again, we insert code at the right points.

```
tos = 0
Stack[tos] = EOF
Stack[++tos] =  root node
Stack[++tos] =  Start Symbol
token = next_token()
repeat
   X = Stack[tos]
   if X is a terminal or EOF then
      if X == token then
         pop X
         token = next_token()
          pop and fill in node
      else error()
   else /* X is a non-terminal */
      if M[X,token] == X -> Y1Y2...Yk then
         pop X
          pop node for X
          build node for each child and
          make it a child of node for X
         push {nk,Yk,nk-1,Yk-1,...,n1,Y1}
      else error()
until X == EOF
```





## A grammar that is not LL(1)

<stmt> ::= if <expr> then <stmt>
 | if <expr> then <stmt> else <stmt>
 | ...

Left-factored:

<stmt> ::= if <expr> then <stmt> <stmt′> | ...
<stmt′> ::= else <stmt> | $\epsilon$

FIRST(<stmt′>) = {$\epsilon$,else}
FOLLOW(<stmt′>) = {else,\$}
LOOKAHEAD(<stmt′> ::= else <stmt>) = {else}
LOOKAHEAD(<stmt′> ::= $\epsilon$) = {else,\$}

On seeing else, conflict between choosing <stmt′> ::= else <stmt> and <stmt′> ::= $\epsilon$

$\Rightarrow$ grammar is not LL(1)!

The fix:

Put priority on <stmt′> ::= else <stmt> to associate else with closest previous then.





## Error recovery

Key notion:

- For each non-terminal, construct a set of terminals on which the parser can synchronize
- When an error occurs looking for $A$, scan until an element of SYNCH($A$) is found

Building SYNCH:

1. $a \in$ FOLLOW($A$) $\Rightarrow a \in$ SYNCH($A$)
2. place keywords that start statements in SYNCH($A$)
3. add symbols in FIRST($A$) to SYNCH($A$)

If we can't match a terminal on top of stack:

1. pop the terminal
2. print a message saying the terminal was inserted
3. continue the parse

(i.e., SYNCH($a$) $= V_t - \{a\}$)