

CA200_Section_03BMoreExercises_Solution_Q1

Question 1

A purchasing agent must decide to accept or reject an incoming shipment of machine parts. The agent wishes to do either of the following:

- a_1 : Accept the shipment
- a_2 : Reject the shipment

The fraction of defective parts in the shipment is either 0.1 or 0.5 with a prior likelihood of each occurring being 0.5. The costs associated with the possible decisions are €1000 if a 0.1 shipment is rejected and €1500 if a 0.5 shipment is accepted. No costs are incurred if a 0.1 shipment is accepted or a 0.5 shipment is rejected. It is possible to test one part from a shipment as a cost of €10.

- (i) What is the optimal decision without sampling?
- (ii) Calculate the Expected Value of Perfect Information.
- (iii) Determine the optimal strategy, that is, what action to take in response to sample outcomes. Show the results of workings on a decision tree.

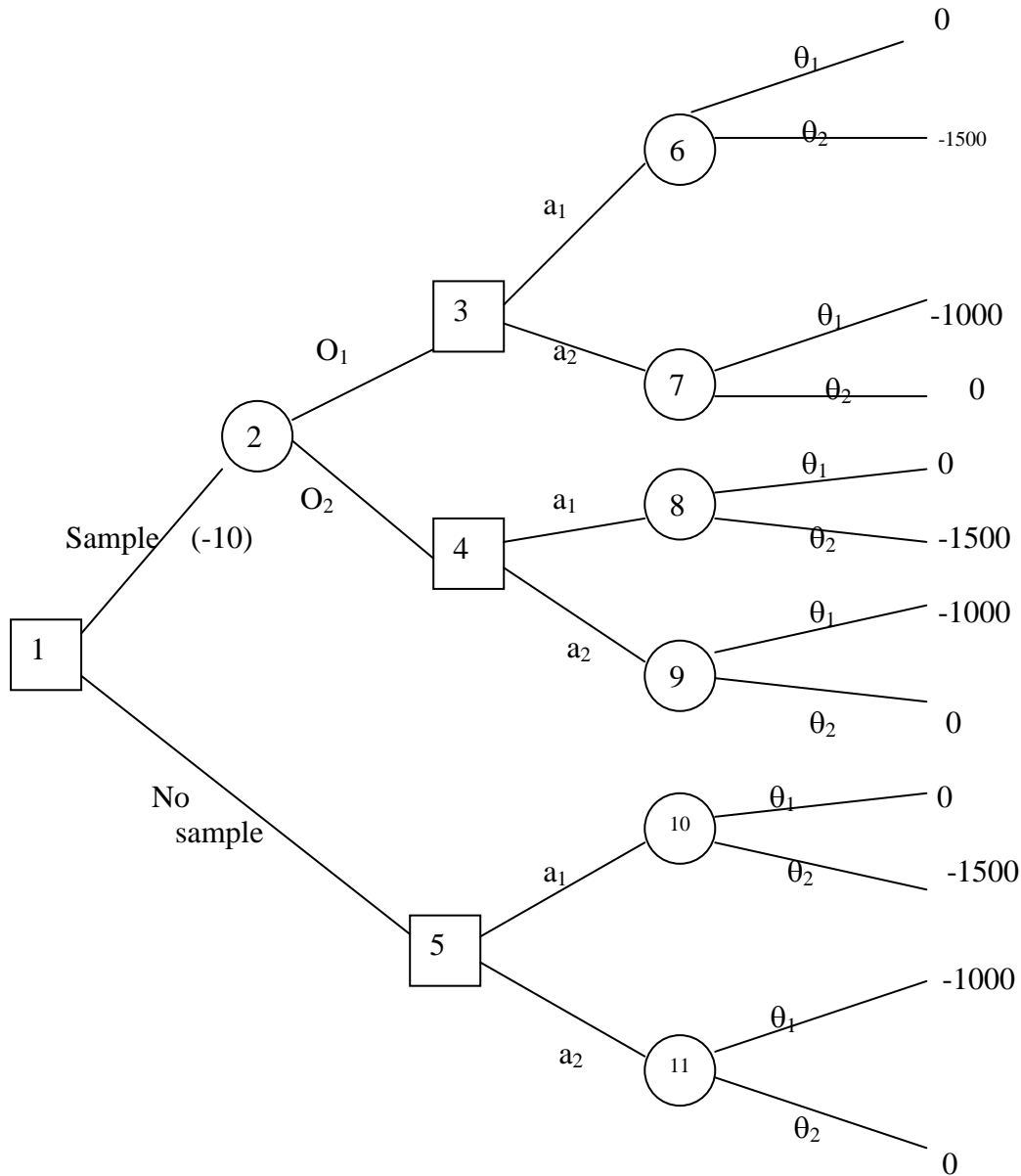
Solution

- (i) $\theta_1 \rightarrow$ shipment with 0.1 defective
 $\theta_2 \rightarrow$ shipment with 0.5 defective
 $E(a_1) = 0(0.5) - 1500(0.5) = -750$
 $E(a_2) = -1000(0.5) + 0(0.5) = -500$
Therefore choose a_2 .

(ii) EVPI

Under perfect information, if θ_1 then choose a_1 with payoff = 0, if θ_2 then choose a_2 with payoff = 0. Therefore $E(\text{payoff})$ under certainty = 0. $E(\text{payoff})$ under uncertainty = -500. Therefore EVPI = 500.

(iii)



By definition $P(\text{faulty}|\theta_1) = 0.1$ and $P(\text{faulty}|\theta_2) = 0.5$
 Let $O_1 =$ tested part is faulty and $O_2 =$ tested part is OK
 $P(\theta_1) = P(\theta_2) = 0.5$

	$P(O_k \theta_j)$	
	O_1	O_2
θ_1	0.1	0.9
θ_2	0.5	0.5

Probability of O_k and θ_j both occurring = $P(O_k \cap \theta_j) = P(O_k | \theta_j)P(\theta_j)$

	O_1	O_2	
θ_1	0.05	0.45	
θ_2	0.25	0.25	therefore $P(O_1) = 0.3$ and $P(O_2) = 0.7$

Probability of shipment θ_1 given test result $O_k = P(\theta_j | O_k) = \frac{P(\theta_j \cap O_k)}{P(O_k)}$

	O_1	O_2
θ_1	0.17	0.64
θ_2	0.83	0.36

Therefore

E.V. at node 6 = -1245

E.V. at node 7 = -170 so E.V. at node 3 = -170

E.V. at node 8 = -540

E.V. at node 9 = -640 so E.V. at node 4 = -540.

Therefore E.V. at node 2 = -429, less 10 for cost of sample.

From (i) E.V. at node 5 = -500

Strategy:

Take sample. If part is faulty, reject shipment. If part is OK, accept shipment.