Q1: Exam results are normally distributed with a mean $\mu = 46$ and a standard deviation $\sigma = 4$. What percentage of students obtain a mark

1. larger than 46
2. larger than 50
3. larger than 40
4. less than 38
5. less than 49
6. between 45 and 49
7. between 50 and 54
8. larger than 56 or less than 40
9. within 1.5 standard deviations from the mean
10. outside of 2.3 standard deviations from the mean?

Some solutions:

In most of these we need to transform to N(0, 1) so as to use the tables using

$Z = (x - \mu)/\sigma = Z = (x - 46)/4$ [*In lectures we sometimes used U instead of Z as that is what appears in the tables but Z is more usual – this is not a big point!*]

1. Here Z = 0 so answer is 50% [*Can be seen from Table 4 but is obvious anyway by symmetry*]
2. Z = 4/4 = 1 => answer is 15.866%
3. Z = -6/4 = -1.5 => answer is 93.319%
4. We need 1 – P(X > 38) = 1 – P(Z > -2) = 1 – 0.97725 = 0.02275 or 2.275%
5. We need 1 – P(X > 49) = 1 – P(Z > 0.75) = 1 – 0.22633 = 0.02275 or 2.275%
6. We need P(45<X<49) = P(X>45) - P(X>49) = p(Z>-0.25) - P(Z>0.75) = 0.59871-0.22663 = 0.37208 or 37.208%
7. Answer is 13.591%
8. We need 1 - P(40<X<56) = … [Answer is 7.3%]
9. P(-1.5 < Z < 1.5) = 1 - 2P(Z > 1.5) = 1 – 2(0.06681) = 0.86638 or 86.638%.
10. 2P(Z > 2.3) = 2(0.01072) = 0.02142 or 2.142%

Q2. Assume the scores on an aptitude test are normally distributed with mean 500 and standard deviation 100.

(a) What is the top 5% cut off point?

(b) What is the middle 40%?

(c) If 1000 new students are to take the exam, predict the number who will score more than 65%

Some solutions:

(a) Find A such that P(X > A) = 0.05. This is the same as P(Z > (A – 500)/100) = 0.05

From Table 4, it follows that (A – 500)/100 =1.65 (approx)

Hence, A – 500 = 165 +> A = 665.

(b) Here we want A such that P(500 - A < X < 500 + A) = 0.4 or 1 - 2P(X > 500 + A) = 0.4 => P(X > 500 +A) = 0.3 => P(Z > A/100) = 0.3 => A/100 = 0.52 => A = 52 that is the middle 40% is between 448 and 512.

(c) This is not that obvious but an acceptable answer would be to assume that the maximum possible mark is $\mu + 3\sigma$ as there is very low probability of being this value. This amounts to 500 + 3(100) = 800 in the present case. Then we are looking for P(X > 0.65(800)) = P(Z > (520 – 500)/100) = P(Z > 0.2) = 0.420074 or 42.074%. This means that 421 new students would score more than 65%.

Another approach would be to take the maximum mark to be 1000 (assuming that the minimum is 0). Then P(X > 650) = P(Z > 1.5) = 0.06681 or 6.681%. However, the first approach perhaps seems more reasonable.

Q3. The heights of women are known to be normally distributed with a mean of 67 inches and a standard deviation of 3. A range of T-shirts are made to fit women of different heights as follows:

Small:	62 to 66 inches
Medium:	66 to 70 inches
Large:	70 to 74 inches

(a) What percentage of the population is in each category?
(b) What percentage is not catered for?
Answer: Nothing very different to Q1.

Q4. A soft drinks machine is regulated so that it discharges an average of 7 ounces per cup. The amount of drink is normally distributed with standard deviation equal to 0.5 ounces.
(a) What is the probability that a cup contains between 6.7 and 7.3 ounces?
(b) How many cups are likely to overflow if 8 ounce cups are used for the next 1000 drinks?
(c) below what value do we get the smallest 25% of the drinks?
Solution: Nothing very new here – will just give answer for (b):
$P(X > 8) = P(Z > (8-7)/0.5) = P(Z > 2) = 0.02275$. Therefore $1000(0.02275) = 23$ cups are likely to overflow.