Semmester 1: Probability

Question 5(b)

An analogue signal received at a detector (measured in microvolts) is normally distributed with a mean of 100 and a variance of 256.

1. What is the probability that the signal will exceed 120 microvolts?
2. What is the probability that it will be between 110 and 120 microvolts?
3. What is the probability that it will be less than 120 microvolts given that it is greater than 110 microvolts?
4. What is the micro-voltage below which 25% of the signals will be?

   Note: The following R output may be of use in solving the above:

```
> pnorm(120, 100, 16)
[1] 0.8943502
> pnorm(110, 100, 16)
[1] 0.7340145
> qnorm(.25, 100, 16)
[1] 89.20816
```

   Alternatively you can use the tables provided.

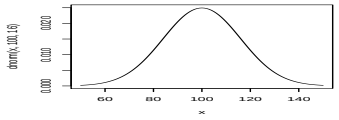
Solution

$$\mu = 100$$
$$\sigma^2 = 256$$
$$\sigma = 16$$

$$X \ N(100, 16)$$

To draw it in R:

```
curve (dnorm(x, 100, 16), 50, 150)
```



(i) What is the probability that the signal will exceed 120 microvolts?

$$P(X > 120)$$

which is

$$1 - P(X \leq 120)$$
$$1 - (pnorm(120, 100, 16)$$
$$1 - 0.8943502 = .10approx$$

(b) $P(110 < X < 120)$ is obtained in R with

```
pnorm(120, 100, 16) - pnorm(110, 100, 16)
```

0.8943502 - 0.7340145
= 0.1603358.

(c) We want

$$P(X < 120|X > 110) = \frac{P\left((X < 120) \cap (X > 110)\right)}{P(X > 110)} = \frac{P(110 < X < 120)}{P(X > 110)}$$

$P(110 < X < 120)$ is obtained in R with

```
pnorm(120, 100, 16) - pnorm(110, 100, 16)
```

which gives 0.1603358.

and

`1-pnorm(110, 100, 16)` $= 0.2659855$. gives $P(X > 110)$.

Then

$$P(X < 120|X > 110) = \frac{0.1603358}{0.2659855} = 0.6027988$$

Area under the standard normal curve from $-\infty$ to z

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6338	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990