EXERCISES 7.1 [1]

1.

2.

3. A binary communication channel carries data as one of two sets of signals denoted by 0 and 1. Owing to noise, a transmitted 0 is sometimes received as a 1, and a transmitted 1 is sometimes received as a 0. For a given channel, it can be assumed that a transmitted 0 is correctly received with probability 0.95, and a transmitted 1 is correctly received with probability 0.75. Also, 70% of all messages are transmitted as a 0. If a signal is sent, determine the probability that:

   (a) a 1 was received;

   (b) a 1 was transmitted given than a 1 was received.

   Solution:

   Let R_0 be the event that a zero is received. Let T_0 be the event that a zero is transmitted.

   Let R_1 be the event that a one is received. Let T_1 be the event that a one is transmitted.

   For (a), the probability that a 1 was received, we note

$$R_1 = (T_0 \cap R_1) \cup (T_1 \cap R_1)$$

   So

$$P(R_1) = P(T_0)P(R_1|T_0) + P(T_1)P(R_1|T_1)$$

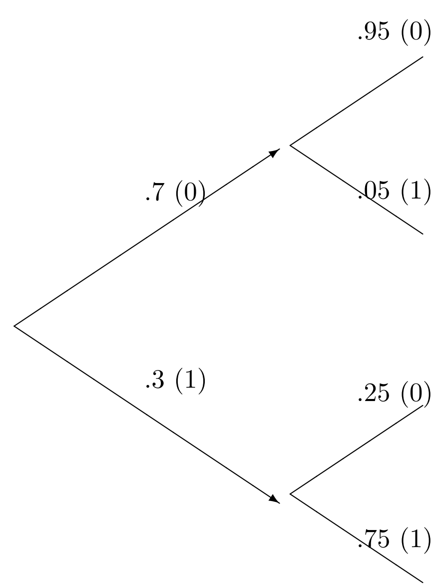
   Therefore

$$P(R_1) = .7 \times .05 + .3 \times .75$$

   A simpler way of doing this is by examining the tree:

[1]Probability with R: An Introduction with Computer Science Applications: Jane M. Horgan, Wiley 2008

Figure 1: Transmit and Receive Probabilities of a Communication Channel



To calculate the probability of receiving a one, we trace the origins of all the ones received, and work along the branches.

For (b) we use Bayes' rule and write

$$P(T_1|R_1) = \frac{P(T_1)P(R_1|T_1)}{P(R_1)}$$

Now:

$$P(T_1)P(R_1|T_1) = .3 \times .75$$

and

$$R_1 = (T_0 \cap R_1) \cup (T_1 \cap R_1)$$

So

$$P(R_1) = P(T_0)P(R_1|T_0) + P(T_1)P(R_1|T_1)$$

Therefore

$$P(R_1) = .7 \times .05 + .3 \times .75 = .26$$

So

$$P(T_1|R_1) = \frac{.7 \times .05}{.7 \times .05 + .3 \times .75} = \frac{.035}{.26}$$

4.

5.

6. A recent study showed that the 10 most common passwords in Ireland and the UK are

123	3.787%
password	3.78%
liverpool	1.82%
letmein	1.76%
123456	1.63%
qwerty	1.41%
charlie	1.39%
monkey	1.1%
arsenal	1.11%
thomas	0.99%

(a) What is the proportion of users with one of these passwords?

```
 .03787 + .0378+.0182 + .0176 + .0163 + .0141 + .0139+ .011 + .0111 + .0099
[1] 0.18777
```

19% approx.

(b) If a hacker knows that your password contains 6 letters and is on this list, what is the probability that yours is selected by her?
6 letters:

```
> .0141+.011+.0099
[1] 0.035
```

$$P(\text{6 letters}|\text{on the list}) = \frac{P(\text{6 letters} \cap \text{list})}{P(\text{list})}$$

```
> .035/.18777
[1] 0.186467
```

7. An e-mail message can travel through one of three server routes. The probability of transmission of error in each of the servers and the proportion of messages that travel each route are shown in the following table. Assume that the servers are independent.

	% messages	% errors
Server 1	40	1
Server 2	25	2
Server 3	35	1.5

(a) What is the probability of receiving an e-mail containing an error?

```
> .4*.01 + .25 *.02 + .35*.015
[1] 0.01425
```

(b) What is the probability that the message will arrive without error?
1- 0.1425 = 0.8575

(c) If a message arrives without error, what is the probability that it was sent through Server 1?

$$P(\text{Server 1}|\text{No error}) = \frac{P(\text{Server 1 } \cap \text{No Error})}{P(\text{No Error})} = \frac{.4 * .99}{.8575} = 0.4618076$$

8. A small software company employs 4 programmers. The percentage of code written by each programmer and the percentage of errors in their code are shown in the following table.

Programmer	% of code written	% of errors in code
1	15	5
2	20	3
3	25	2
4	40	1

Code is selected at random.

(a) Before the code is examined, what is the chance that it was written by

i. Programmer 1? .15
ii. Programmer 2? .20
iii. Programmer 3? .25
iv. Programmer 4? .40

(b) Suppose the code is examined and found to be free of errors. After this examination, what is the probability that it was written by Programmer 1?

$$P(\text{Prog 1}|\text{no error}) = \frac{P(\text{Prog 1} \cap \text{error})}{P(\text{no error})}$$

$$P(\text{Prog 1} \cap \text{no error}) = .15 * .95 = .1425$$

P (no error):

```
> .15*.95 + .20 *.97 + .25 * .98 + .40 * .99
[1] 0.9775
```

$$P(\text{Prog 1}|\text{no error}) = .1425/.9775 = 0.1457428$$

$P(\text{Prog 2}|\text{no error})$

```
> .20*.97/.97775
[1] 0.1984147
```

$P(\text{Prog 3}|\text{no error})$

```
> .25*.98/.97775
[1] 0.2505753
```

$P(\text{Prog 4}|\text{no error})$

```
> .40*.99/.97775
[1] 0.4050115
```