■ Example 2: Mountain Profiles

We are given a set of shapes and we want to order them in such a way as to bring out their similarities and differences. A geologist might desire such an ordering because, when combined with other geological information, it could reveal the kinds of erosional processes that cause the shape of mountains to evolve with time.

Technically, the ordering is achieved by a method called "Factor Analysis" (use of the important ***Singular Value Decomposition***), the details of which which we won't go into. The example illustrates some of the available means for setting up arrays of data, of displaying them graphically and of manipulating them.

(a) Setting up the data:

```
peaks = {{0, 1.2, 2.4, 3.3, 4.4, 5.5, 4.4, 3.3, 2.4, 1.2, 0},
   {0, 2.2, 3.2, 3.8, 4.5, 5, 4.5, 3.8, 3.2, 2.2, 0},
   {0, .7, 1.1, 3.4, 4.6, 5.4, 4.6, 3.4, 1.1, .7, 0},
   {0, 1, 3.2, 5.4, 5.5, 5.1, 4.4, 3.8, 3.3, 2.1, 0},
   {0, 2.2, 3.3, 3.9, 4.4, 5.1, 5.6, 5.7, 3.6, 1, 0},
   {0, 5.4, 5.4, 5.4, 4.9, 4.9, 4.9, 3.3, 2.3, 1, 0},
   {0, 1, 2.3, 3.3, 5, 5, 5, 5.6, 5.6, 5.6, 0},
   {0, .01, .02, .6, 2.3, 5.5, 2.3, .6, .02, .01, 0},
   {0, .7, 1.1, 2.4, 3.4, 5.6, 3.4, 2.4, 1.1, .7, 0},
   {0, .5, 1, 3.3, 5.4, 3.3, 1.1, .5, .5, .05, 0},
   {0, .05, .07, .07, 1.1, 3.3, 5.8, 3.5, 1.1, .6, 0},
   {0, .01, .02, .02, .6, .6, 1.1, 2.4, 5.6, 2.4, 0},
   {0, 2.6, 5.6, 2.5, 1.2, .7, .6, .02, .01, .01, 0},
   {0, .01, .02, .8, 2.4, 5.6, 5, 4.9, 3.9, 2.8, 0}};
```

(b) Displaying the data in tabular form:

```
TableForm[peaks]
```

0	1.2	2.4	3.3	4.4	5.5	4.4	3.3	2.4	1.2
0	2.2	3.2	3.8	4.5	5	4.5	3.8	3.2	2.2
0	0.7	1.1	3.4	4.6	5.4	4.6	3.4	1.1	0.7
0	1	3.2	5.4	5.5	5.1	4.4	3.8	3.3	2.1
0	2.2	3.3	3.9	4.4	5.1	5.6	5.7	3.6	1
0	5.4	5.4	5.4	4.9	4.9	4.9	3.3	2.3	1
0	1	2.3	3.3	5	5	5	5.6	5.6	5.6
0	0.01	0.02	0.6	2.3	5.5	2.3	0.6	0.02	0.01
0	0.7	1.1	2.4	3.4	5.6	3.4	2.4	1.1	0.7
0	0.5	1	3.3	5.4	3.3	1.1	0.5	0.5	0.05
0	0.05	0.07	0.07	1.1	3.3	5.8	3.5	1.1	0.6
0	0.01	0.02	0.02	0.6	0.6	1.1	2.4	5.6	2.4
0	2.6	5.6	2.5	1.2	0.7	0.6	0.02	0.01	0.01
0	0.01	0.02	0.8	2.4	5.6	5	4.9	3.9	2.8

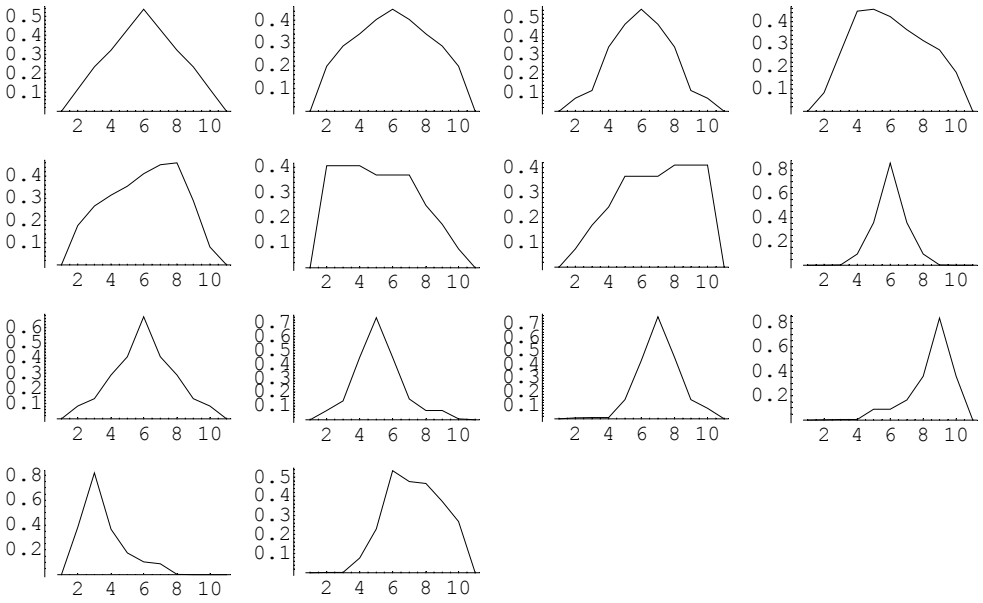
(c) "Normalizing" the data (we are here interested in shape and need to remove differences in size or height):

```
shapes = Table[peaks[[i]] / Sqrt[Apply[Plus, peaks[[i]]^2]], {i, 1, 14}];
```

(d) Plotting the data (as an array of separate plots):

```
shapeplots = Table[ListPlot[shapes[[i]], PlotJoined → True,
    AxesOrigin → {0, 0}, DisplayFunction → Identity], {i, 1, 14}];
```

```
Show[
 GraphicsArray[{{shapeplots[[1]], shapeplots[[2]], shapeplots[[3]], shapeplots[[4]]},
   {shapeplots[[5]], shapeplots[[6]], shapeplots[[7]], shapeplots[[8]]},
   {shapeplots[[9]], shapeplots[[10]], shapeplots[[11]], shapeplots[[12]]},
   {shapeplots[[13]], shapeplots[[14]]}}]]
```



- GraphicsArray -

(e) Processing the data (using "singular value decomposition (SVD)"):

- First interchange rows and columns (i.e. transpose):

```
s = Transpose[shapes];
```

- Apply SVD:

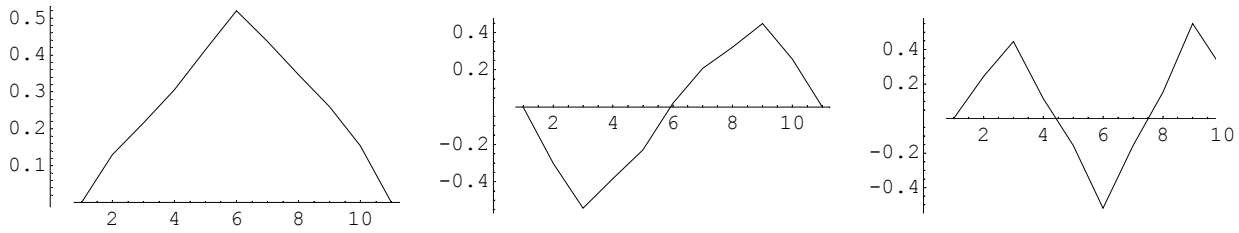
```
{u, md, v} = SingularValues[s]; original = Transpose[u].DiagonalMatrix[md].v;
```

*- The elements of "***u***" contain, in order, the most important elements present in our shapes. The amount of each "element" present in each of our 14 shapes is given b the array "***factors***", calculated as follows:*

```
factors = DiagonalMatrix[md].v;
```

(f) Plots of the three most important elements - "average or mean", "skewness" and "sharpness" :

```
Show[GraphicsArray[{{ListPlot[-u[[1]], PlotJoined → True, AxesOrigin → {0, 0},
     DisplayFunction → Identity], ListPlot[u[[2]], PlotJoined → True,
     AxesOrigin → {0, 0}, DisplayFunction → Identity], ListPlot[u[[3]],
     PlotJoined → True, AxesOrigin → {0, 0}, DisplayFunction → Identity]}}]]
```



- GraphicsArray -

(g) ***Quantifying similarities and differences in the 14 shapes in terms of the amount of each element present*** :

- Numerical values of the factors for the three main elements:

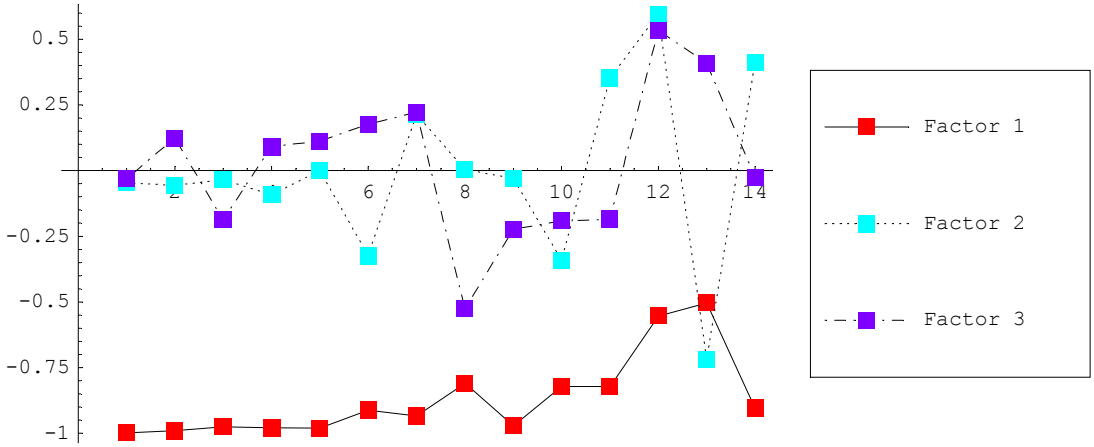
```
TableForm[Transpose[{factors[[1]], factors[[2]], factors[[3]]}]]
```

-0.998017	-0.0461721	-0.0308295
-0.990042	-0.05727	0.123197
-0.975633	-0.0344173	-0.188062
-0.978581	-0.0927172	0.0913235
-0.980208	8.36817×10^{-6}	0.111891
-0.912011	-0.3236	0.176016
-0.933633	0.214145	0.222077
-0.809742	0.00383675	-0.526039
-0.971035	-0.0295082	-0.22325
-0.821442	-0.341496	-0.191841
-0.821079	0.350945	-0.185711
-0.553763	0.594267	0.533457
-0.503594	-0.718841	0.409327
-0.904574	0.41247	-0.0258314

We see that, except for mountains 12 and 13, there is not difference in the amount of the first element. However, the amounts of the other two elements do vary considerably. These data are plotted below.

```
<< Graphics`MultipleListPlot`
```

```
MultipleListPlot[{factors[[1]], factors[[2]], factors[[3]]}, PlotJoined → True,
 SymbolShape → {PlotSymbol[Box, 4], PlotSymbol[Box, 4], PlotSymbol[Box, 4]},
 SymbolStyle → {Hue[0], Hue[.5], Hue[.75]},
 PlotLegend → {"Factor 1", "Factor 2", "Factor 3"}]
```



- Graphics -

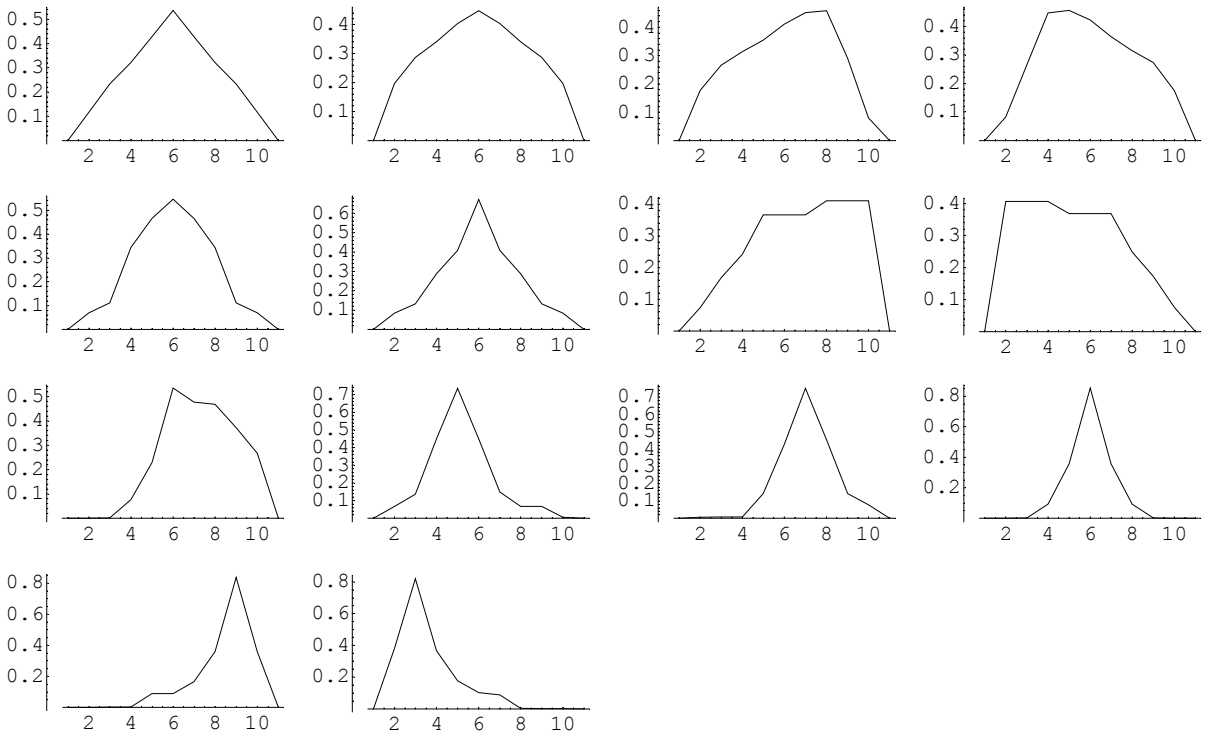
(h) RESULTS

(I) Mountains in increasing order of factor 1 ("average"):

```
Sort[factors[[1]]]
```

```
{-0.998017, -0.990042, -0.980208, -0.978581, -0.975633, -0.971035, -0.933633,
 -0.912011, -0.904574, -0.821442, -0.821079, -0.809742, -0.553763, -0.503594}
```

```
Show[
 GraphicsArray[{{shapeplots[[1]], shapeplots[[2]], shapeplots[[5]], shapeplots[[4]]},
   {shapeplots[[3]], shapeplots[[9]], shapeplots[[7]], shapeplots[[6]]},
   {shapeplots[[14]], shapeplots[[10]], shapeplots[[11]], shapeplots[[8]]},
   {shapeplots[[12]], shapeplots[[13]]}}]]
```



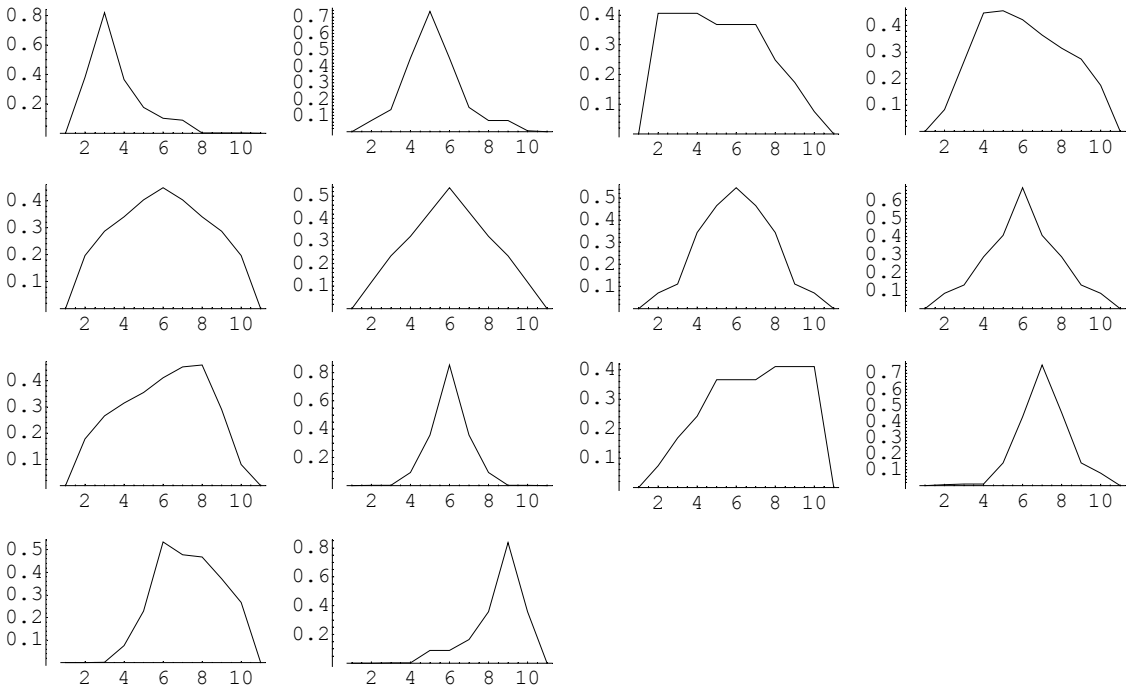
- GraphicsArray -

(II) Mountains in increasing order of factor 2 ("skewness"):

```
Sort[factors[[2]]]
```

```
{-0.718841, -0.341496, -0.3236, -0.0927172, -0.05727, -0.0461721, -0.0344173,
 -0.0295082, 8.36817×10^-6, 0.00383675, 0.214145, 0.350945, 0.41247, 0.594267}
```

```
Show[
 GraphicsArray[{{shapeplots[[13]], shapeplots[[10]], shapeplots[[6]], shapeplots[[4]]},
   {shapeplots[[2]], shapeplots[[1]], shapeplots[[3]], shapeplots[[9]]},
   {shapeplots[[5]], shapeplots[[8]], shapeplots[[7]], shapeplots[[11]]},
   {shapeplots[[14]], shapeplots[[12]]}}]]
```



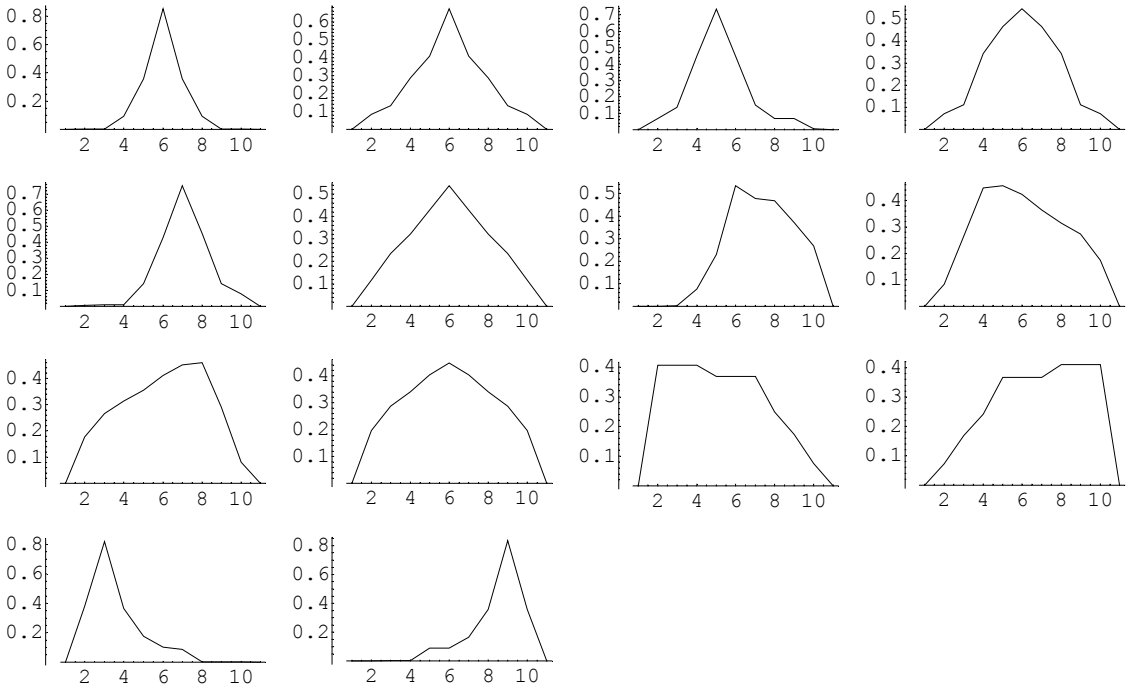
- GraphicsArray -

(II) Mountains in increasing order of factor 3 ("sharpness"):

```
Sort[factors[[3]]]
```

```
{-0.526039, -0.22325, -0.191841, -0.188062, -0.185711, -0.0308295, -0.0258314,
 0.0913235, 0.111891, 0.123197, 0.176016, 0.222077, 0.409327, 0.533457}
```

```
Show[
 GraphicsArray[{{shapeplots[[8]], shapeplots[[9]], shapeplots[[10]], shapeplots[[3]]},
   {shapeplots[[11]], shapeplots[[1]], shapeplots[[14]], shapeplots[[4]]},
   {shapeplots[[5]], shapeplots[[2]], shapeplots[[6]], shapeplots[[7]]},
   {shapeplots[[13]], shapeplots[[12]]}}]]
```



- GraphicsArray -