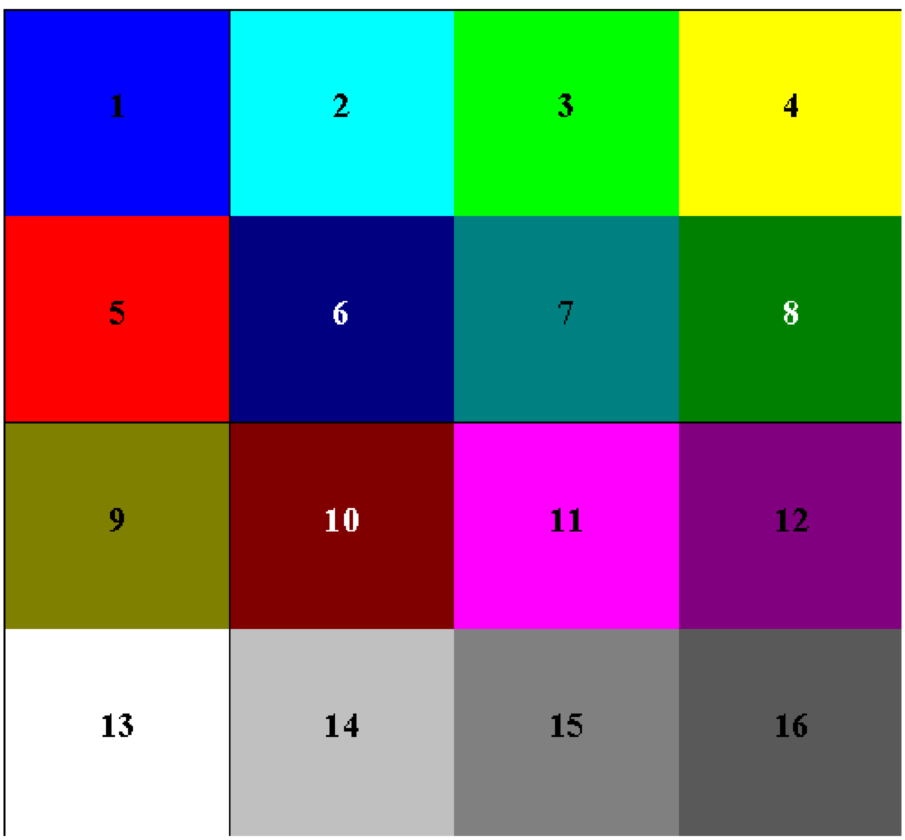
■ Example 3: Acoustic Tomography

In general, "Tomography" is the process of forming images of the interior of an object from measurements made along rays passed through that object ("tomo" comes from the Greek word for "slice").

(a) Definition & Model of the problem:
Suppose that a wall is assembled from a rectangular array of bricks and that each brick is composed of a different kind of clay. The following illustrates the case of a square array of 16 bricks. The bricks are numbered for reference, as shown.



If the acoustic velocities of the different clays differ, one might attempt to distinguish the different kinds of bricks by ***measuring the travel time (T) of sound across the various rows and columns of bricks*** in the wall. This would give us 8 measurements in all (4 rows and 4 columns).

To ***model*** this situation, we will assume that each brick is uniform and that the travel time of sound across each brick is proportional to the width and height of the brick. The proportionality factor is the brick's slowness s_i. Thus, we have a total of 16 slowness parameters for the above diagram.

Assuming that the bricks are all of unit height and width we have the following 8 ***observation equations***:

row 1: $T_1 = s_1 + s_2 + s_3 + s_4$ ..... row 4: $T_4 = s_{13} + s_{14} + s_{15} + s_{16}$
col. 1: $T_5 = s_1 + s_5 + s_9 + s_{13}$ ..... Col. 4: $T_8 = s_4 + s_8 + s_{12} + s_{16}$

The problem is to estimate values of the 16 unknown slowness parameters from the 8 time measurements.

NOTE: Technically, this is an example of a "***MIXED-DETERMINED PROBLEM***". It is partly underdetermined (more unknowns than data values) but also can be shown to contain redundant information.

<u>(b) What are we going to illustrate here?</u>:
- Set up an artificial situation where we know the "true" values of the slowness parameters.
- Test a method of solving mixed-determined problems by seeing how well it can reproduce the "true" values
- An important part of the solution method is that it requires an initial (or "a priori") guess to be made of the slowness parameter values

<u>N.B. Do not be concerned with the detail of the algorithms used</u>.

<u>(c) Technical detail, re setting up "observation equations" - Case of 100 = 10 X 10 bricks</u>:

```
nnum = 20; mnum = 100;

g = Table[0, {i, 1, nnum}, {j, 1, mnum}];

Do[g[[i, j]] = 1, {i, 1, nnum / 2}, {j, 1 + (nnum / 2) * (i - 1), (nnum / 2) * i}];
```

```
Do[g[[i + nnum / 2, j]] = 1, {i, 1, nnum / 2}, {j, i, mnum, nnum / 2}];

{u, md, v} = SingularValues[N[g]];
```

(d) Setting up artificial "true" situation (mostly slowness parameters of 1/10 but one "patch" of 1/12):

```
mexact = Table[1 / 10, {i, 1, mnum}]; Do[mexact[[nnum / 2 + 1 + k]] = 1 / 12, {k, 1, 3}];
Do[mexact[[2 * nnum / 2 + 1 + k]] = 1 / 12, {k, 1, 3}];
Do[mexact[[3 * nnum / 2 + 1 + k]] = 1 / 12, {k, 1, 3}];

mexact2d = Partition[mexact, nnum / 2];

TableForm[N[mexact2d]]
```

0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
0.1	0.0833333	0.0833333	0.0833333	0.1	0.1	0.1	0.1	0.1
0.1	0.0833333	0.0833333	0.0833333	0.1	0.1	0.1	0.1	0.1
0.1	0.0833333	0.0833333	0.0833333	0.1	0.1	0.1	0.1	0.1
0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1

(e) Using artificial "true" slowness values to generate the "measured" travel times:

```
dexact = g.mexact;
```

(f) Setting up "a priori" information (constant slowness of "1/10"):

```
mapriori = Table[1 / 10, {i, 1, mnum}];
```

(g) Applying the solution method to estimate the slowness parameters:

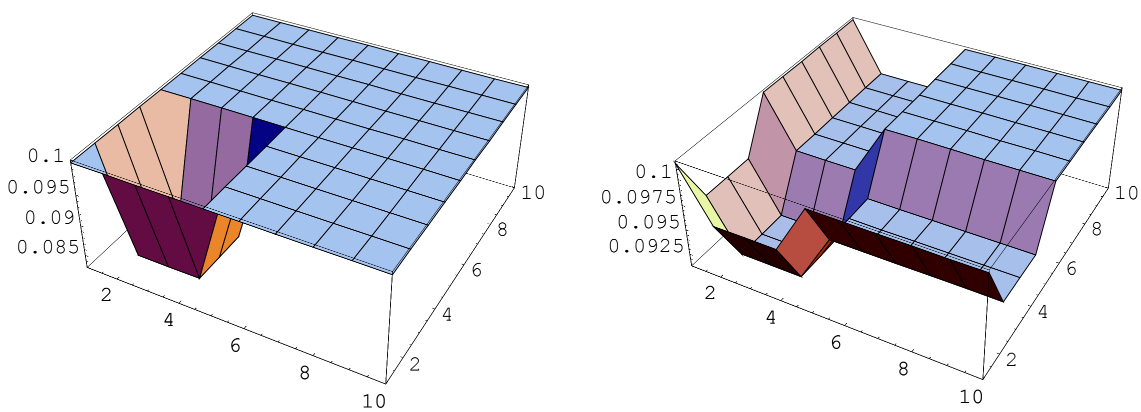
```
mest = mapriori + PseudoInverse[N[g]].(dexact - g.mapriori);

mest2d = Partition[mest, nnum / 2];
```

```
TableForm[N[mest2d]]
```

0.1015	0.0965	0.0965	0.0965	0.1015	0.1015	0.1015	0.1015
0.0965	0.0915	0.0915	0.0915	0.0965	0.0965	0.0965	0.0965
0.0965	0.0915	0.0915	0.0915	0.0965	0.0965	0.0965	0.0965
0.0965	0.0915	0.0915	0.0915	0.0965	0.0965	0.0965	0.0965
0.1015	0.0965	0.0965	0.0965	0.1015	0.1015	0.1015	0.1015
0.1015	0.0965	0.0965	0.0965	0.1015	0.1015	0.1015	0.1015
0.1015	0.0965	0.0965	0.0965	0.1015	0.1015	0.1015	0.1015
0.1015	0.0965	0.0965	0.0965	0.1015	0.1015	0.1015	0.1015
0.1015	0.0965	0.0965	0.0965	0.1015	0.1015	0.1015	0.1015
0.1015	0.0965	0.0965	0.0965	0.1015	0.1015	0.1015	0.1015

(h) Graphical comparison of "true" and "estimated" slowness parameters:

```
Show[GraphicsArray[{{ListPlot3D[mexact2d, DisplayFunction → Identity],
    ListPlot3D[mest2d, DisplayFunction → Identity]}}]]
```




```
- GraphicsArray -
```