Closed-form approximation as a basis for a light-weight Radiative Transfer Model

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Introduction

– Briefly review Radiative Transfer and its relevance to Earth Observation.
– Outline characteristics of Radiative Transfer Models generally.
– Summarise some work on closed-form approximations, and how it might be used as the basis for a computationally light-weight Radiative Transfer Model.

Radiative Transfer (RT)

- Radiative Transfer concerns the propagation of radiation through media which absorb, scatter and/or emit photons.
- Radiative Transfer Equation describes the radiative transfer process mathematically.
- The optical domain (ultra-violet to thermal infrared) within the Earth’s atmosphere-surface system is of particular interest for Earth Observation (EO).

Radiative Transfer Models (RTM)

- “Radiative transfer modelling creates the link between the remote sensing instrument and the target being observed. As such, radiative transfer tools have a key role in:
  - The design of remote sensing instruments …
    - Simulate remote sensing instrument observations
    - Perform sensitivity/feasibility/trade-off analysis …
    - Develop future mission end-to-end simulators
  - The development & testing of inversion algorithms for the retrieval of geophysical variables (Level 2 algorithms)”

“Towards a Generic Radiative Transfer Model ,,,”, ESA ITT 2007

Radiative Transfer Models (RTM)

- Radiative Transfer involves complex interactions between photons and the scattering and absorption centres comprising the attenuating medium.
- This complexity prevents obtaining an exact solution to the radiative transfer equation (RTE), in general.
- Many radiative transfer models have been developed that vary considerably in their accuracy and computational speed. For example,
  - Semi-empirical Monte Carlo models have been developed that are highly accurate and comprehensive, but demanding of computing resources.
  - Simpler, faster but less accurate models can be derived on the basis of simplifying assumptions (e.g. isotropy, homogeneity, decoupling of absorption and scattering).

Classical approach to RT (Chandrasekhar)

- Chandrasekhar's "Radiative Transfer" is the definitive, classical text.
- Chandrasekhar defined a numerical procedure for approximate solution of the radiative transfer equation at any point within a medium.
- His formulation of the **emergent radiation** from a medium is of particular interest for this presentation, however.

Classical approach to RT (Chandrasekhar)

He found that **emergent radiation** (I) from a plane-parallel, semi-infinite medium can be expressed in terms of "H-functions" that are themselves solutions of non-linear integral equations. For example, for isotropic media,

I. *Isotropic scattering*:

$$I(0,\mu,\varphi;\mu_0,\varphi_0) \equiv I(0,\mu,\mu_0) = \frac{1}{4}\frac{\varpi_0}{\mu+\mu_0}H(\mu)H(\mu_0)\mu_0 F,$$

where $H(\mu)$ is defined in terms of the characteristic function

$$\Psi(\mu) = \text{constant} = \tfrac{1}{2}\varpi_0.$$

The H-functions can be calculated numerically but here we are interested in "closed-form" approximations (many such have been published for the isotropic case).

ω_0 ε [0, 1] is the single scattering albedo, = 0 or 1 for no scattering or complete (conservative) scattering, respectively.

μ_0 and μ define the inward and outward radiation directions, respectively.

Classical approach to RT (Chandrasekhar)

- The same essential structure applies for less simple cases. For example,

II. *Scattering in accordance with the phase function* $\varpi_0(1+x\cos\Theta)$:

$$I(0,\mu,\varphi;\mu_0,\varphi_0)=\frac{1}{4}\frac{\varpi_0}{\mu+\mu_0}\{\psi(\mu)\psi(\mu_0)-x\,\phi(\mu)\phi(\mu_0)+$$

$$+x[(1-\mu^2)^{\frac{1}{2}}H^{(1)}(\mu)][(1-\mu_0^2)^{\frac{1}{2}}H^{(1)}(\mu_0)]\cos(\varphi_0-\varphi)\}\mu_0 F,$$

where $\psi(\mu)=H(\mu)(1-c\mu)$ and $\phi(\mu)=q\mu H(\mu)$

Note: For this case, one could model different radiative transfer regimes by choice of parameters ω_0 & x which, of course, vary with wavelength.

Inhomogeneity could be simulated by varying ω_0 & x with surface position.

Classical approach to RT (Chandrasekhar)

- The general anisotropic case is summarised as follows (by Lenoble – If S is known (2.49), the emergent radiation (1.31) follows):

$$I^{+}(0;+\mu,\phi) = \frac{1}{4\pi\mu} S(\tau_1,0;\mu,\phi;\mu_0,\phi_0)\,\pi F, \qquad (1.31)$$

$$S(\mu,\mu_o;\phi) = S^{o}(\mu,\mu_o)+2\sum_{m=1}^{N} S^{m}(\mu,\mu_o)\cos m\phi, \qquad (2.49)$$

Classical approach to RT (Chandrasekhar)

Each azimuthal component may be expressed in terms of auxiliary functions $\phi_i^m(\mu)$ as

$$\left(\frac{1}{\mu}+\frac{1}{\mu_o}\right)S^m(\mu,\mu_o) = \bar{\omega}_o \sum_{i=m}^{N} \beta_i(-1)^{i+m}\,\phi_i^m(\mu)\,\phi_i^m(\mu_o)\,\frac{(i-m)!}{(i+m)!}\;. \qquad (2.50)$$

Each $\phi_i^m(\mu)$ may be expressed by means of an H-function H^m:

$$\phi_i^m(\mu) = \sqrt{(2m)!}\;q_i^m(\mu)\,P_m^m(\mu)\,H^m(\mu)\,, \qquad (2.51)$$

where

$$H^m(\mu) = 1+\mu\,H^m(\mu)\int_0^1 \frac{\Psi^m(\mu')}{\mu+\mu'}\,H^m(\mu')\,d\mu'\,, \qquad (2.55)$$

Closed-form approximations for classical approach to RT

- It is shown (JQSRT article, 2005) that one can obtain approximations in closed-form for H-functions, of the form,

$$H^*[\mu, m] = \frac{1}{1 - t^*[m] f[\mu, m]}.$$

If greater accuracy is required, particularly near the conservative case of pure scattering, improved approximations are possible of the forms,

$$H^{\mathrm{L}}[\mu, m] = H^*[\mu, m] - \beta[m]\mu.$$

or

$$H^{\mathrm{Q}}[\mu, m] = H^*[\mu, m] - \beta[m]\mu - \gamma[m]\mu^2$$

Closed-form approximations for a computationally light-weight RTM?

- It appears that the foregoing results may be used as the basis for a computationally light-weight (i.e. no numerical RTE solving) but quite accurate model for radiative transfer.
- Use of closed-form rather than numerical approximations may enable easier interpretation of model outputs.
- In principle, it should be possible to model different radiative transfer regimes, including different degrees of anisotropy, by suitable choice of ω_0 and higher order parameters.
- In general, these parameters depend on position (inhomogeneity) & wavelength. So, an approach based on closed-form approximations offers the possibility of alleviating an otherwise considerable computational burden.

Future Work

- In principle, there are no basic obstacles to implementing the proposed RT model, although there are technical issues particularly regarding boundary conditions (there is existing literature that can be drawn on).
- Several simplifying assumptions were made in the above (e.g. plane-parallelism, no polarization). It is desirable to see whether some of these can be relaxed.
- A particular assumption is of semi-infinite rather than finite layers. The latter are investigated by Chandrasekhar, X- and Y- functions replacing H-functions, but it is not yet clear if one can obtain useful closed-form approximations for these functions.
- It would be useful to investigate the approximations with more mathematical rigour as they may fall within a broader class of similar problems (involving non-linear integral equations).
- It would be of interest to investigate whether the closed-form approximations apply, or have analogues in the broader field of transport theory where one may also have $\omega_0 > 1$ (transport in a multiplying medium).

(A few) References

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