

## 1. Project Title

Closed form approximation-based approach to Anisotropic Radiative Transfer, including Inversion [CARTI]

## 2. Proposer & Funding sought

**Proposer:** Dr. W.G. Tuohey, School of Computing, Dublin City University.

**Funding:** Funding not immediately available. One funding possibility is for prospective postgraduate researchers to apply to the “Embark Initiative Postgraduate Research Scholarship Scheme” administered by IRCSET (Irish Research Council for Science, Engineering & Technology - [http://www.ircset.ie/grant\\_schemes/postgrad.html](http://www.ircset.ie/grant_schemes/postgrad.html))

## 3. Project Description

### 3.1 Abstract

#### SCIENTIFIC/SPECIALIST AUDIENCE:

Building on recent results of the author, based on an iterative scheme using symbolic computation, the research aims to develop an approximate closed-form model for the emergent radiation from a plane-parallel, optically anisotropic medium. This model will then be used in the inverse problem of determining optical properties of the medium from measurements of the emergent radiation. The model will be encapsulated in a portable and open software component to enhance its usability within larger systems. Extensions to more general inverse problems and integral equations will be investigated.

#### GENERAL AUDIENCE:

Essentially, inverse theory is concerned with extracting useful inferences about the world from physical measurements. In this research the physical measurements are of the emerging light from a medium. Certain objects of interest have characteristic optical signatures, particularly in how they absorb and scatter light at different wavelengths, and this makes it possible in principle to make inferences about a medium's constituents from measuring the emergent light. A model is needed to describe a medium optically and such modelling is the main focus of the research. Application areas include environmental monitoring and photomedicine.

### **3.2 Scientific Content**

#### 1. What is the question that this proposal addresses?

How to develop a novel closed-form approximation for the radiation emergent from an optically anisotropic medium?

How can this approximation

- yield new theoretical and practical insights
- be used to invert remotely sensed measurements to determine scattering albedo (scattering to total attenuation ratio), and related parameters
- be the basis for an operational software component, usable over a range of areas of application
- be characterized (esp. convergence properties) and generalized in mathematical terms?

#### 2. Why is this problem significant?

Radiative transfer arises in many fields including astrophysics [1], terrestrial remote sensing and photomedicine. Light travelling through a medium may be absorbed or scattered, and the inverse problem of radiative transfer is to infer properties of the medium from measurements of the light passing through it. This is feasible because at least some of the materials in the medium may have particular optical ‘signatures’. For example, dissolved yellow pigments occurring in lakes and inshore waters have high absorption in blue wavelengths and low absorption in red [2]. In diagnostic photomedicine [3], incorporating the interaction of light with biological tissues, information may be acquired from scattering as well as absorption – for example, red light penetrates more deeply into biological tissues because it is not strongly absorbed by blood and because it tends to scatter more. In general, absorption and scattering contain chemical and physical information, respectively [4]. There are various scattering ‘regimes’, including isotropic, Rayleigh (molecular) and Mie (particles of dimension greater than the light wavelength). As a specific illustration, [5] notes that ‘several human tissues have properties that depend not only on location but also on direction’ [i.e. are anisotropic], ‘brain white matter’ and ‘muscle fibre’ being cited as examples.

In general, radiative transfer within a medium is described by a first order linear integro-differential equation. However, for plane parallel problems, Chandrasekhar [1] showed that the emergent radiance from a medium can be expressed in terms of

functions which satisfy certain non-linear integral equations. In the case of semi-infinite media, these functions are called H functions and satisfy equations of the form

$$H[\mu, m] = 1 + \mu H[\mu, m] \int_0^1 \frac{\psi[\mu', m] H[\mu', m] d\mu'}{\mu + \mu'}$$

There are corresponding functions, called X and Y functions, for media of finite optical thickness. In the past, this approach has been considered intractable for complex anisotropic problems; for example, “The method of X, Y, and H functions can, in principle, be applied to scattering problems with arbitrary phase functions. In practice, the amount of algebraic manipulations is too great ...”[6]. However, recently [7] the author of this proposal has developed, with the aid of *Mathematica* for symbolic computation, a closed form approximation for H functions in the general anisotropic case. The core of this proposal is to build on this result to establish explicit approximate models for the emergent radiation.

In general, a fundamental fact in inverse theory is “... that problems based on actual measurements can never have unique solutions” [8]. However, it is a considerable advance if a soundly based model has been identified to fit the measurements – this is the intended contribution of the proposed research.

### 3. How will the question be addressed?

The proposed research splits into a number of work packages (Figure 1). The intended work for each work package is summarised below.

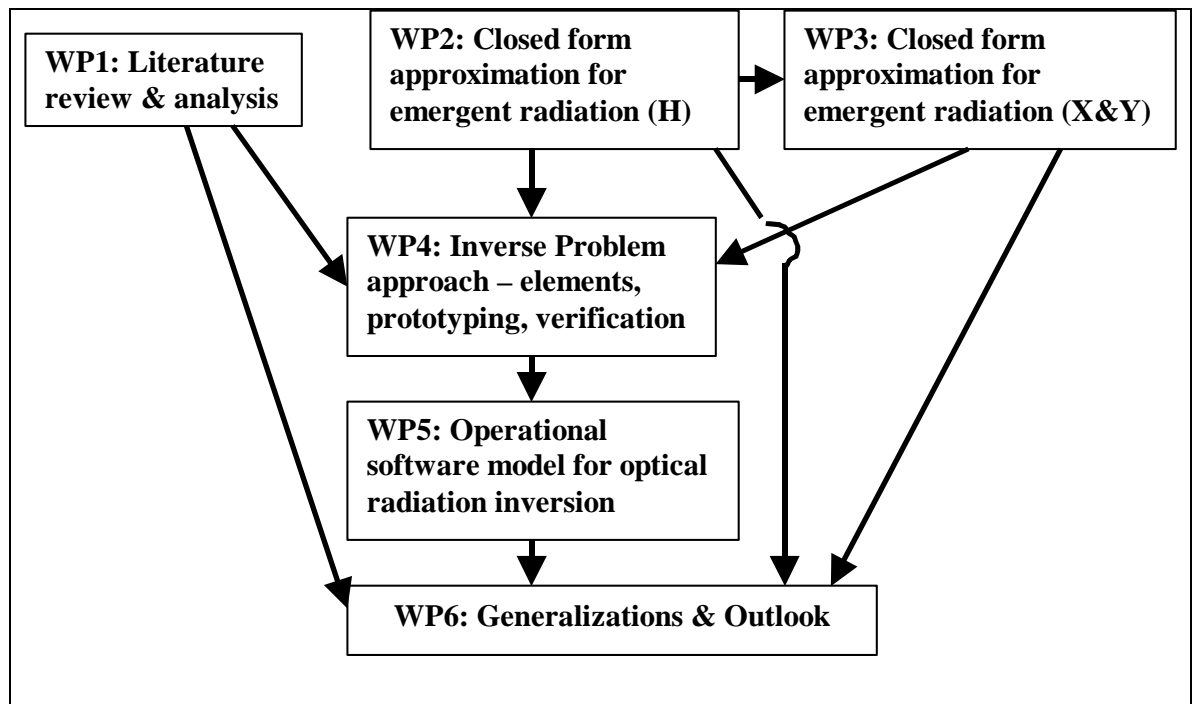


Figure 1: Indicative Work Breakdown Structure and Sequencing

WP1: Literature review and analysis

This will be wide-ranging, particularly covering as many as possible of the diverse fields of application of radiative transfer.

WP2: Closed form approximation for emergent radiation (H – semi infinite media):

In [9], a computational procedure is outlined which, combined with this proposal author's new H function results [7], holds out the promise that a "closed form" approximation for the emergent radiation can be developed. In the first place, explicit expression(s) will be established for the emergent radiation corresponding to scattering phase functions of up to 2 harmonics (linearly anisotropic & Rayleigh). Then, guided by these particular cases, an attempt will be made to establish the general anisotropic case. In addition, the particular cases are of intrinsic interest – even in the isotropic case it is of interest, in the application of airborne remote sensing of lakes and adjacent lands, to derive an explicit closed form approximation for the "visible reflectance" (and related concepts) and to compare this with existing formulations.

WP3: Closed form approximation for emergent radiation (X & Y –finite optical media):

Analogous X & Y function approximations, to the H function approximations of [7], will be sought following the same approach. This is likely to be quite challenging but would be very worthwhile. A follow-on step, similar to WP2, would be to seek approximations for both the emergent and transmitted radiation.

WP4: Inverse problem approach – elements, prototyping, verification:

Reference [10] (with its supporting artifacts) includes a prototype simulation, for the isotropic case only, of the radiation inversion process to extract the scattering albedo parameter from (simulated) measurements of the reflectance from a medium. The tasks in this work package are mainly concerned with extending that simulation and making it more realistic. Thus, in particular, the simulation will be extended to include anisotropic models (using the results of the previous two WPs). Another important task is to make explicit how to handle non-zero boundary conditions, the original results of [1] being for atmospheres where zero boundary conditions were appropriate. The realism of the simulation will be enhanced by improving the modelling of measurement errors including investigation of the characteristics of representative instruments and observation conditions. Finally, as an initial validation and

assessment of the enhanced simulation model, simulated measurements will be replaced by some actual data.

WP5: Operational software model for optical radiation inversion:

In order to facilitate incorporation of the research results into wider systems, as well as to make them readily available to other researchers, it is proposed to develop an operational software model in the form of a reusable component [11]. Taking the simulation of WP4 as a prototype, this will include systematically defining software requirements for the radiative transfer model, the methods of inversion, the interpretation of results, etc. It will also include the definition of verification, including system tests, and extensive validation including against published research. While it will be possible to re-use particular methods or functions from the WP4 simulation, the overall system design will be thoroughly re-worked. The standard UML methodology (use cases, class model, etc through to implementation and deployment) will be applied in developing the component.

WP6: Generalizations & Outlook:

Mathematical research will be performed to better characterize the iterative method, especially in terms of convergence properties, used in [7] to derive the approximate H-function closed form. Also, it will be investigated whether, and if so to what extent, this iterative approach can be applied to a wider class of non-linear integral equations of the Hammerstein type. Finally, major areas for future research and development will be identified.

4. What is the recent record of accomplishment in research that argues for project success?

The proposer has been working in this research area for a number of years, especially since 2002. The initial motivation was to enhance understanding of the radiative transfer process in an application involving airborne remote sensing of lakes and adjacent lands. Publications [7] and [10] have resulted from the work together with corresponding software programs.

Also, [http://www.ercim.org/publication/Ercim\\_News/enw61/tuohey.html](http://www.ercim.org/publication/Ercim_News/enw61/tuohey.html) (O'Mongain, E. & Tuohey, W.G. (2005), "Extracting Maximum Information from Airborne Remote Sensing", ERCIM News No. 61) is a brief informal account of some aspects of the work. In related research, a Masters in Bioinformatics student has recently completed a practicum report (minor dissertation) under the proposer's

supervision (McBride, R. (Sep. 2005), “Vegetation indices and their potential to detect eutrophication”).

#### 5. What is the value of this research to the people of Ireland?

The research will add to the level of Irish expertise in the important field of inverse theory and methods. In terms of applications, it may contribute to improvements in environmental remote sensing and in diagnostic photomedicine.

### **4. References**

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