DUBLIN CITY UNIVERSITY

SEMESTER ONE REPEAT EXAMINATION 2004

MODULE:	CA540 Computational Biology
COURSE:	M. Sc. in BioInformatics
YEAR:	1
EXAMINERS:	Mr. P. Cunningham, Dr. Martin Crane, (ext 8974)
TIME ALLOWED:	2 hours
INSTRUCTIONS:	Attempt three questions. All questions carry equal marks.
REQUIREMENTS:	Mathematical Tables, Graph Paper

THE USE OF PROGRAMMABLE OR TEXT STORING CALCULATORS IS EXPRESSLY FORBIDDEN

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QUESTION 1

(a) Explain fully what is meant by the *eigendecomposition* of a matrix $\mathbf{A}$?

[5 marks]

How does the stability of a system of difference equations given by

$$\mathbf{u}_n = \mathbf{A}\mathbf{u}_{n-1}$$

depend on the eigenvalues of the matrix $\mathbf{A}$?

[6 marks]

(b) Define what is meant by a Leslie matrix and describe in words what is the role of the dominant eigenvalue and corresponding eigenvector of a Leslie matrix.

[10 marks]

In a certain species of insect population, the following rules for breeding and survival of females apply:

- $\frac{1}{16}$ survive their first birthday and live into a second year.
- $\frac{1}{4}$ of these survive their second birthday and live into their third year.
- By the end of the third year, all the original insects are dead.
- No insects are born until a female survives into its second year, when an average of seven new insects are produced. This average drops to six in the third year of a females life.

Set up a Leslie matrix for the population.

[6 marks]

Given that there is a single positive eigenvalue, show that this has the value $\frac{3}{4}$ and, hence or otherwise, show that the population is condemned to extinction.

[6 marks]

QUESTION 2

(a) Write down the equation for unconstrained (Malthusian) growth. What are the assumptions behind this model?

[10 marks]

(b) Starting with the above equation you have written down, derive an expression for the doubling time of a species undergoing such growth.

[9 marks]

(c) According to Bruckman's book on Cancer:

> with most cancers, you could only begin to detect a lump when the number of cancer cells reached approximately one billion (10^9).

Assuming the initial cancer cell starts off on January 1 2000 and reproduces once every month, what is the earliest that it would become detectable?

[7 marks]

Assuning that 100 billion cells weigh about 125g and that a human body can tolerate at most 2.5 kg of tumour, how long roughly before such a tumour becomes lethal?

[7 marks]

QUESTION 3

(a) Using (if you like) the logistic growth model as an example, describe the key difference between discrete and continous models of change.

[7 marks]

(b) Starting with the continuous Logistic Growth model:

$$\frac{dN}{dt} = kN\left(1 - \frac{N}{K}\right)$$

(where N is the number in the population), Derive, using partial fractions or otherwise, the general solution:

$$N(t) = \frac{K}{\left(\frac{K}{N_0} - 1\right)e^{-kt} + 1}$$

where N_0 is the initial value of N)

[15 marks]

(c) Sketch the curve of $N(t)$ against t for the following initial conditions:

- $N_0 > K/2$
- $N_0 < K/2$

[6 marks]

and describe qualitatively the shape of the curves.

[5 marks]

QUESTION 4

(a) Describe qualitatively the chemostat apparatus under the following headings:

- purpose
- how bacterial production is maintained
- how growth differs from logistic model

[15 marks]

(b) The non-dimensional form of the chemostat equations can be shown to be

$$\begin{aligned}\frac{dn}{d\tau} &= \alpha_1 \left(\frac{c}{1+c}\right) n \quad - n \\ \frac{dc}{d\tau} &= -\left(\frac{c}{1+c}\right) n \quad - c + \alpha_2\end{aligned}$$

(where τ, n, c are non-dimensional time, nutrient concentration and culture chamber concentration, respectively and α_1, α_2 are dimensionless parameters). Derive an expression for the Jacobian of the above system.

[4 marks]

(c) Determine the steady states and say whether they are stable or unstable.

[6 marks]

(d) From the steady states say with reasons what are sensible ranges of values for α_1 and α_2.

[8 marks]

QUESTION 5

(a) Briefly describe with examples, the three types of interaction model we covered in the course.

[12 marks]

(b) Starting with the symbiosis model for logistic growth with positive interaction:

$$\begin{aligned}\dot{x} &= \mu_1 x \left(1 - \frac{x}{K_1} \quad + c_{12}\frac{y}{K_1}\right) \\ \dot{y} &= \mu_2 y \left(1 - \frac{y}{K_2} \quad + c_{21}\frac{x}{K_2}\right)\end{aligned}$$

(where a dot indicates differentiation w.r.t. time, $c_{12}, c_{21}, \mu_1, \mu_2$ are positive constants of interaction, K_1, K_2 are the different carrying capacities and x, y are the numbers of species respectively), show by non-dimensionalizing as follows:

$$u_1 = \frac{x}{K_1} \quad u_2 = \frac{y}{K_2}, \quad \tau = \mu_1 t$$

that this may be reduced to non-dimensional form.

[9 marks]

(c) Derive the Jacobian matrix of the above symbiosis equation in dimensionless form.

[4 marks]

(d) Find the four steady states and, using the Jacobian matrix, determine whether they are stable or unstable.

[8 marks]