DUBLIN CITY UNIVERSITY

SEMESTER ONE EXAMINATION 2004

MODULE:	CA540 Computational Biology
COURSE:	M. Sc. in BioInformatics
YEAR:	1
EXAMINERS:	Mr. P. Cunningham, Dr. Martin Crane, (ext. 8974)
TIME ALLOWED:	2 hours
INSTRUCTIONS:	Attempt three questions. All questions carry equal marks.
REQUIREMENTS:	Mathematical Tables, Graph Paper

THE USE OF PROGRAMMABLE OR TEXT STORING CALCULATORS IS EXPRESSLY FORBIDDEN

DO NOT TURN OVER THIS PAGE UNTIL INSTRUCTED TO DO SO.

QUESTION 1

Explain fully what is meant by the *eigendecomposition* of a matrix $\mathbf{A}$?

[5 marks]

How does the stability of a system of difference equations given by

$$\mathbf{u}_n = \mathbf{A}\mathbf{u}_{n-1}$$

depend on the eigenvalues of the matrix $\mathbf{A}$?

[6 marks]

Define what is meant by a Leslie matrix and describe in words the role of the dominant eigenvalue and corresponding eigenvector of a Leslie matrix.

[10 marks]

In a population of an imaginary organism that lives for two years, the average number of births for one-year-olds is 1/3 and that for two-year-olds is 4. The survival probability from one to two years old is 2/3. Death is certain after 3 years.

1. Set up a Leslie matrix for the population.

[6 marks]

2. Find the long-term growth rate and the stable age distribution for this population.

[6 marks]

QUESTION 2

(a) In terms of the continuous Logistic Growth model with Harvesting:

$$\frac{dN}{dt} = kN\left(1 - \frac{N}{K}\right) - H$$

(where N is the number in the population), briefly describe, in qualitative terms the role of each term on the right hand side.

[9 marks]

(b) Using appropriate non-dimensionalisation, show how the above equation can be reduced to:

$$\frac{dy}{dt} = ky\,(1 - y) - h$$

[8 marks]

(c) Hence find the steady states p_+, p_- of the above system.

[6 marks]

(d) By sketching a graph of $\dot{y}$ V y, say briefly what will happen for the initial conditions:

- $y(0) < p_-$
- $y(0) > p_-$

[10 marks]

QUESTION 3

(a) Define clearly the following terms:

- *Steady state*
- *Stable state*
- *Asymptotically stable state*

[6 marks]

(b) For a system of equations:

$$\frac{d\mathbf{x}}{dt} = \mathbf{F}(\mathbf{x}) = \begin{pmatrix} f(x_1, x_2) \\ g(x_1, x_2) \end{pmatrix}$$

show that for a point $\hat{\mathbf{x}}$ close to a steady state $\bar{\mathbf{x}}$, we can say that

$$\frac{d\hat{\mathbf{x}}}{dt} \approx \mathbf{A}\mathbf{x}$$

where $\mathbf{A}$ is the Jacobian Matrix of $\mathbf{F}$

[10 marks]

(c) Define the Jacobian Matrix $\mathbf{A}$ term-by-term

[4 marks]

(d) If the Jacobian Matrix, given by

$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

has the characteristic equation:

$$\lambda^2 - p\lambda + q = 0$$

Define p and q

[7 marks]

(e) Give the range of values of p, q for stability.

[6 marks]

QUESTION 4

(a) Briefly describe, with examples, three types of non-linear interaction model.

[12 marks]

(b) Starting with the Lotka-Volterra Predator-Prey model:

$$\begin{aligned} \dot{x} &= ax \quad -by \\ \dot{y} &= -cy \quad +dxy \end{aligned}$$

(where a dot indicates differentiation w.r.t. time, a, b, c, d are positive constants and x, y are the numbers of prey and predators respectively), show using non-dimensionalisation how this may be reduced to

$$\begin{aligned} \frac{du}{d\tau} &= u(1 \quad -v) \\ \frac{dv}{d\tau} &= \gamma v(u \quad -1) \end{aligned}$$

(where γ is some dimensionless parameter)

[8 marks]

(c) Derive a general expression for the Jacobian matrix of a Lotka-Volterra system.

[5 marks]

(d) Find the two steady states of the Lotka-Volterra system and say whether they are stable or unstable.

[8 marks]

QUESTION 5

(a) Broadly classify infectious diseases.

[2 marks]

(b) Give two reasons why it is not feasible to model infectious diseases as predator-prey systems.

[4 marks]

(c) Give the assumptions behind the Susceptibles-Infectives- Removed (SIR) model of infectious diseases

[4 marks]

(d) Starting with the equations for the SIR model:

$$\begin{aligned} \dot{S} &= -\beta IS \\ \dot{I} &= \beta IS \quad -\nu I \\ \dot{R} &= \qquad\quad \nu I \end{aligned}$$

(where a dot over a letter indicates differentiation w.r.t. time and β, ν are the so-called infection and removal rates), derive an expression for I in terms of S.

[10 marks]

(e) Roughly graph a phase-plane plot of I against S (the position of $\rho = \nu/\beta$ should be clearly visible)

[7 marks]

(f) Describe *qualitatively* what happens for $S < \rho$ and $S > \rho$.

[6 marks]