DUBLIN CITY UNIVERSITY

SEMESTER ONE EXAMINATION 2005

MODULE:	CA540 Computational Biology
COURSE:	M. Sc. in BioInformatics
YEAR:	1
EXAMINERS:	Mr. P. Cunningham, Dr. Martin Crane, (ext. 8974)
TIME ALLOWED:	3 hours
INSTRUCTIONS:	Attempt **four** questions. If you answer more than four, please indicate which questions you wish to be marked. All questions carry equal marks.
REQUIREMENTS:	Mathematical Tables, Graph Paper

THE USE OF PROGRAMMABLE OR TEXT STORING CALCULATORS IS EXPRESSLY FORBIDDEN

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QUESTION 1

Explain fully what is meant by the *eigendecomposition* of a matrix $\mathbf{A}$?

[3 marks]

How does the stability of a system of difference equations given by

$$\mathbf{u}_n = \mathbf{A}\mathbf{u}_{n-1}$$

depend on the eigenvalues of the matrix $\mathbf{A}$?

[4 marks]

Define what is meant by a Leslie matrix and describe in words the role of the dominant eigenvalue and corresponding eigenvector of a Leslie matrix.

[8 marks]

We are given that a female population is divided into 3 equal age groups and that the average no. of female offspring for the first, second and third age groups are 1,4.5 and 3.2 respectively. If the proportion of the first and second age groups which survive to the next age group is 0.9 and 0.7 respectively

1. Set up a Leslie matrix for the population.

[2 marks]

2. Find the long-term growth rate and the stable age distribution for this population.

[8 marks]

QUESTION 2

A factor that can affect the distribution of genes in a population is *mutation*. This happens when external factors (e.g. background radiation) cause an allele A to change to an allele α (say). The rate at which mutation occurs is usually very small (at fraction 10^{-5} per generation) and more commonly acts to change a dominant gene into a recessive gene. A big question is to find the growth of proportion of the mutants in the population. We assume random mating, equal survival and, in addition, we assume that A-alleles mutate to α-alleles at a rate μ per generation.
(a) Show, that, without mutation but with random mating and equal survival, at the end of the generation $k+1$, the numbers with each genotype, $\mathcal{N}_{k+1}()$ (in terms of the total number of fertilized eggs at the start of that generation N^*_{k+1}) is given by:

$$\begin{aligned}
\mathcal{N}_{k+1}(AA) &= r\left[P_k(A)\right]^2 N^*_{k+1} \\
\mathcal{N}_{k+1}(A\alpha) &= 2rP_k(A)P_k(\alpha)N^*_{k+1} \\
\mathcal{N}_{k+1}(\alpha\alpha) &= r\left[P_k(\alpha)\right]^2 N^*_{k+1}
\end{aligned}$$

where r is the survival fraction from the beginning to the end of the generation and $P()$ is the proportion of a particular allele in the population.

[10 marks]

(b) If μ is the mutation rate and

$$P_{k+1}(\alpha) = \frac{no.\ of\ \alpha - alleles\ +\ \mu \times (\ no.\ of\ A - alleles)}{total\ no.\ of\ alleles\ in\ gene\ pool}$$

show that the proportion of α-alleles at the end of year $k+1$ is given by:

$$P_{k+1}(\alpha) = (1-\mu)P_k(\alpha) + \mu$$

[10 marks]

(c) Hence derive an expression for $P_{k+1}(\alpha)$ in terms of the initial proportion of α-alleles P_0

[5 marks]

QUESTION 3

(a) In terms of the continuous Logistic Growth model with Harvesting:

$$\frac{dN}{dt} = kN\left(1 - \frac{N}{K}\right) - H$$

(where N is the number in the population), briefly describe, in qualitative terms the role of each term on the right hand side.

[7 marks]

(b) Using appropriate non-dimensionalisation, show how the above equation can be reduced to:

$$\frac{dy}{dt} = ky\,(1-y) - h$$

[5 marks]

(c) Hence find the steady states p_+, p_- of the above system.

[6 marks]

(d) By sketching a graph of $\dot{y}$ V y, say briefly what will happen for the initial conditions:

- $y(0) < p_-$
- $y(0) > p_-$

[7 marks]

QUESTION 4

(a) Write down the equation for unconstrained (Malthusian) growth. What are the assumptions behind this model?

[8 marks]

(b) Starting with the equation you have written down, derive an expression for the doubling time of a species undergoing such growth.

[7 marks]

(c) According to Bruckman's book on cancer:

> with most cancers, you could only begin to detect a lump when the number of cancer cells reached approximately one billion (10^9).

Assuming the initial cancer cell starts off on January 1 2000 and reproduces once every month, what is the earliest that it would become detectable?

[5 marks]

Assuning that 100 billion cells weigh about 125g and that a human body can tolerate at most 2.5 kg of tumour, how long roughly before such a tumour becomes lethal?

[5 marks]

QUESTION 5

(a) Briefly describe, with examples, three types of non-linear interaction model.

[10 marks]

(b) Starting with the Lotka-Volterra Predator-Prey model:

$$\begin{aligned} \dot{x} &= ax \quad -bxy \\ \dot{y} &= -cy \quad +dxy \end{aligned}$$

(where a dot indicates differentiation w.r.t. time, a, b, c, d are positive constants and x, y are the numbers of prey and predators respectively), show using non-dimensionalisation how this may be reduced to

$$\begin{aligned} \frac{du}{d\tau} &= u(1 \quad -v) \\ \frac{dv}{d\tau} &= \gamma v(u \quad -1) \end{aligned}$$

(where γ is some dimensionless parameter)

[6 marks]

(c) Derive a general expression for the Jacobian matrix of a Lotka-Volterra system.

[3 marks]

(d) Find the two steady states of the Lotka-Volterra system and say whether they are stable or unstable.

[6 marks]

QUESTION 6

(a) Give two reasons why it is not feasible to model infectious diseases as predator-prey systems.

[4 marks]

(b) Give the assumptions behind the Susceptibles-Infectives- Removed (SIR) model of infectious diseases

[4 marks]

(c) Starting with the equations for the SIR model:

$$\begin{aligned} \dot{S} &= -\beta IS \\ \dot{I} &= \beta IS \quad -\nu I \\ \dot{R} &= \nu I \end{aligned}$$

(where a dot over a letter indicates differentiation w.r.t. time and β, ν are the so-called infection and removal rates), derive an expression for I in terms of S.

[8 marks]

(d) Roughly graph a phase-plane plot of I against S (the position of $\rho = \nu/\beta$ should be clearly visible)

[5 marks]

(e) Describe *qualitatively* what happens for $S < \rho$ and $S > \rho$.

[4 marks]