



SEMESTER TWO EXAMINATIONS 2006

Module Code: CA151
Introductory Statistics

Programmes: B.Sc. in Mathematical Sciences
B.Sc. in Financial Mathematics

Year: 1

Examiner & Ext No: Patricia Gunning Ext 8449
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Time Allowed: 2 Hours

Instructions: Please answer any **THREE** questions. All questions carry equal marks.

Requirements: Statistical Tables

THE USE OF PROGRAMMABLE OR TEXT STORING CALCULATORS IS EXPRESSLY FORBIDDEN.

PLEASE DO NOT TURN OVER THIS PAGE UNTIL INSTRUCTED TO DO SO

PLEASE NOTE THAT WHERE A CANDIDATE ANSWERS MORE THAN THE REQUIRED NUMBER OF QUESTIONS, THE EXAMINER WILL MARK ALL QUESTIONS ATTEMPTED AND THEN SELECT THE HIGHEST SCORING ONES.

Q1. (a) State and prove Bayes' theorem. (10)

(b) State the Multiplication Law of Probability. (3)

(c) If it can be assumed that a person is equally likely to be born on any of the 7 days of the week, what is the probability that

- i) among 4 persons, at least two were born on the same day of the week?
ii) among 4 persons, all were born on different days of the week? (8)

(d) A software company employs 3 programmers in one of their subsidiaries. The percentage of code written by each programmer and the percentage of errors in their code are shown in the following table:

Programmer	Percentage of Code Written	Percentage of Errors in Code
A	55%	2%
B	35%	3%
C	10%	5%

i) Suppose a programmer is selected at random. **Before** the code is examined, what is the chance that it was written by:
Programmer A?
Programmer B?
Programmer C? (3)

ii) Suppose that the code is examined and found to contain errors. **After** this examination, what is the chance that it was written by:
Programmer A?
Programmer B?
Programmer C? (9)

Q2. (a) Define the Poisson distribution and the exponential distribution. (2)

(b) Derive the mean of the Poisson distribution. (5)

(c) In what circumstances can the Poisson distribution be used to approximate the Binomial distribution? (4)

(d) In a large set of tax returns to be validated by a tax inspector, it is known from past experience that approximately 2% of them are in error. If the tax inspector selects 6 returns from this set, what is the probability that at least 2 are in error? (5)

If, instead, the tax inspector selects 60 returns, what is the probability that at least 2 are in error? (5)

- Q2 (e) The average rate of calls a receptionist receives is 3 per minute. If it can be assumed that the number of calls per minute has a Poisson distribution, calculate the probability that:
- more than 3 calls will arrive in one minute.
 - no calls are received in one minute.
 - at least 40 seconds will elapse between any two calls.
- (12)

- Q3 (a) In the context of statistical inference, **briefly** explain what is meant by the following terms:
- the sampling distribution of the mean.
 - the standard error of the mean.
 - an unbiased estimator.
- (6)

- (b) 200 students from each of two different courses sat the same exam (i.e. a total of 400 students). The 200 students in course A scored a mean grade of 68 with a standard deviation of 20, while those in course B had a mean score of 73 with a standard deviation of 18. Is there any reason to believe that students in course B are significantly better than those in course A? Use the 5% significance level.
- (12)

- (c) A simple random sample of size 10 drawn from a normal distribution yielded a sample mean of 100 and a sample variance of 64.
- Obtain a 99% confidence interval for the true mean of the population.
 - What would the interval be if the population variance were known to be $\sigma^2 = 81$?
 - If it is required to be 95% confident that the sample mean is to be correct to within 2 euro of the true mean, what sample size is necessary? Use $\sigma^2 = 81$.
- (15)

- Q4 (a) In an attempt to determine if men and women differ with respect to their preferences for different types of music, a sample of 250 females and 250 males were asked to specify their preferred type. The following results were obtained:

	Classical	Pop	Country
Female	83	97	70
Male	77	63	110

- Test the hypothesis that there is a difference in the preference pattern of females and males at the 5% significance level.
- (16)

Q4 (b) Exam results are normally distributed with a mean $\mu = 50$ and a standard deviation $\sigma = 10$. A pass is awarded to students who obtain a mark greater than or equal to 40%. First class honours is awarded to students who obtain a mark greater than or equal to 70%. What is the total percentage of students who will either fail or achieve first class honours? (8)

(c) It has been observed that a certain measurement is a normal random variable of which 25% of values are less than 10.5 cm and 5% are greater than 14.0 cm. What are the mean and variance of the measurements? (9)

Q5. Five pharmaceutical firms had the following profits and research expenditures:

Profit (Thousands of Euro)	Research Expenditure (Thousands of Euro)
100	40
110	40
150	60
180	75
220	95

- i) Determine the Least Squares Regression line of profit on research expenditure for the above data. (14)
- ii) Determine the coefficient of correlation of the above data. (14)
- iii) What would you expect profit to be if 70,000 euro was spent on research? (5)