



SEMESTER TWO REPEAT EXAMINATIONS 2006

Module Code: CA151
Introductory Statistics

Programmes: B.Sc. in Mathematical Sciences
B.Sc. in Financial Mathematics

Year: 1

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Time Allowed: 2 Hours

Instructions: Please answer any **THREE** questions. All questions carry equal marks.

Requirements: Statistical Tables

THE USE OF PROGRAMMABLE OR TEXT STORING CALCULATORS IS EXPRESSLY FORBIDDEN.

PLEASE DO NOT TURN OVER THIS PAGE UNTIL INSTRUCTED TO DO SO

PLEASE NOTE THAT WHERE A CANDIDATE ANSWERS MORE THAN THE REQUIRED NUMBER OF QUESTIONS, THE EXAMINER WILL MARK ALL QUESTIONS ATTEMPTED AND THEN SELECT THE HIGHEST SCORING ONES.

Q1. (a) State and prove the Addition Law of Probability. (6)

(b) A secretary receives application forms for a current vacancy and has to check them for completeness before further processing is carried out on them. About one in every 20 is incomplete.

i) What is the probability that at least one incomplete form will occur in two successive forms that the secretary checks?

ii) In a batch of 100, what is the probability that at least one incomplete form will occur?

iii) How many form checks would result in a 10% chance of coming across an incomplete form? (18)

(c) A company that manufactures tennis balls operates three shifts each day. Based on past experience, it is known that a percentage of the balls produced will be defective. The following table shows the percentage produced on each shift and the percentage of defectives in each shift:

Shift	Percentage Produced	Percentage of Defectives
1	30%	10%
2	50%	15%
3	20%	20%

If a tennis ball is chosen at random and found to be defective, what is the probability that it was produced on the first shift? (9)

Q2 (a) Derive the expectation and variance of the binomial distribution. (13)

(b) Define the geometric distribution and derive the expression for the cumulative distribution. (6)

(c) On the bus route that a student takes to college, there are 5 traffic lights between the stop the student boards the bus and the stop the student embarks. Assume that the probability that a light will be red is 0.25. Assuming independence, what is the probability that at least three traffic lights will show a red light before the student gets off the bus? (7)

(d) Auditions for a new musical will last on average 20 minutes per person. The director in charge of the audition knows from past experience that the probability that a person is suitable for the part is 0.8. Due to time constraints, the first person auditioned that the director deems suitable is selected. If the talent for any one person is independent of another, what is the probability that the director will have found the person for the part in exactly one hour after starting the auditions? (7)

- Q3 (a) In the context of hypothesis testing, distinguish between:
Type 1 and type 2 errors. (2)
- (b) An electrical firm claims to produce bulbs with a mean lifetime of 900 hours. The standard deviation of the process is known to be 50. A sample of 40 gave a mean 885 hours. Test whether the claim is exaggerated at the 5% significance level. (8)
- (c) The scores in a statistics examination are normally distributed with a mean of 55% and a standard deviation of 15. Students who achieve between 40% and less than 55% are awarded a pass. Those who get between 55% and less than 65% are awarded a grade of 2.2. Those who get between 65% and less than 75% are awarded a grade of 2.1 while those who achieve 75% or more are awarded first class honours.
i) Estimate the proportion of students at each award level.
ii) What number would be expected to obtain each award level in a class of 500? (12)
- (d) A market research company wants to estimate the proportion of people who will buy their new product. The company wishes to have 95% confidence that their estimate will be within plus or minus 0.02 of the true proportion who will buy the new product. What sample size is needed? (6)
- (e) A sample size calculated in (d) was taken and it was found that 62% of those interviewed would buy the new product. Estimate the 90% confidence interval for the true proportion of people who will buy the new product. (5)
- Q4 (a) Define the expected value $E(X)$ and the variance $V(X)$ for both discrete and continuous random variables. (2)
- (b) If $Y = a + bX$ where X and Y are discrete random variables and a and b are constants, derive an expression for
i) $E(Y)$
ii) $V(Y)$ (4)

Q4 (c) For the following joint distribution

i) Calculate the marginal distribution for each of X and Y . (4)

ii) Indicate if X and Y are independent, justifying your answer. (7)

X	Y	
	1	2
0	.06	.04
1	.30	.20
2	.24	.16

(c) A count of the number of burglaries per week in a given city over a period of 2 years (104 weeks) produced the following frequency distribution:

Number of Burglaries	0	1	2	3	4+
Frequency	61	30	8	5	0

Use these data to estimate the mean number of burglaries per week and to test the hypothesis that the number of burglaries follows a Poisson distribution. Use the 5% level of significance. (16)

Q5. In a skating competition, 5 contestants are awarded a score out of a possible 30 marks by 5 judges. The following table shows the number of votes each contestant polled from the general public and the corresponding average score awarded by the judges:

Number of Public Votes (in thousands)	Average Score of the Judges
120	17
200	19
450	25
580	25
820	28

i) Determine the Least Squares Regression line of public votes on the average score awarded by the judges for the above data. (14)

ii) Determine the coefficient of correlation of the above data. (14)

iii) How many votes from the public should a skater who got an average score of 22 from the judges expect to receive? (5)