

GEOMETRIC DISTRIBUTION

EXAMPLES:

1. Toss a coin repeatedly. Let X = number of tosses to first head
2. It is known that in a large accounting population 10% of accounts are in error. Accounts are inspected until first account in error is encountered. Let X = number of inspections to obtain first account in error.
3. Roll a die until first 5.
Let X = number of roll until first 5.
4. Observing single births until a girl.
Let X = number of observations to first girl.

GEOMETRIC DISTRIBUTION

Conditions:

1. An experiment consists of repeating trials until first success.
2. Each trial has two possible outcomes;
 - (a) A success with probability p
 - (b) A failure with probability $q = 1 - p$.
3. Repeated trials are independent.

X = number of trials to first success

X is a **GEOMETRIC RANDOM VARIABLE**.

PDF:

$$P(X = x) = q^{x-1}p; \quad x = 1, 2, 3, \dots$$

CDF:

$$P(X \leq x) = P(X = 1) + P(X = 2) \cdots P(X = x)$$

$$= p + qp + q^2p \cdots + q^{x-1}p$$

$$= p[1 - q^x]/(1 - q)$$

$$= 1 - q^x$$

Example:

From past experience it is known that 3% of accounts in a large accounting population are in error.

- What is the probability that 5 accounts are audited before an account in error is found?

$$\begin{aligned}P(X = 5) &= P(\text{1st 4 correctly stated})P(\text{5th in error}) \\ &= (0.97^4)(0.03)\end{aligned}$$

- What is the probability that the first account in error occurs in the first five accounts audited?

$$\begin{aligned}P(X \leq 5) &= 1 - P(\text{First 5 correctly state}) \\ &= 1 - 0.97^5\end{aligned}$$