

## CA215: Sample $\lambda$ -calculus question

The following are expressions in the *untyped*  $\lambda$ -calculus. Reduce the expressions as far as possible. Remember that abstraction is right-associative and that application is left-associative.

- (i)  $(\lambda a.a)b$
- (ii)  $(\lambda a.aa)b$
- (iii)  $(\lambda a.x)b$
- (iv)  $(\lambda a.aa)(\lambda b.bb)$
- (v)  $(\lambda a.\lambda b.abc) p q$
- (vi)  $(\lambda p.((\lambda q.pqr)c))d$
- (vii)  $(\lambda ab.a)(\lambda p.p)$
- (viii) NOT TRUE
- (ix) AND TRUE FALSE
- (x) OR TRUE FALSE

where

- TRUE :=  $(\lambda x.(\lambda y.(x)))$
- FALSE :=  $(\lambda x.(\lambda y.(y)))$
- NOT :=  $(\lambda p.(p \text{ FALSE } \text{ TRUE}))$
- AND :=  $(\lambda p.(\lambda q.(p q \text{ FALSE})))$
- OR :=  $(\lambda p.(\lambda q.(p \text{ TRUE } q)))$