

CA215: Sample λ -calculus solution

- (i) $(\lambda a.a)b =_{\beta} b$
- (ii) $(\lambda a.aa)b =_{\beta} bb$
- (iii) $(\lambda a.x)b =_{\beta} x$
- (iv) $(\lambda a.aa)(\lambda b.bb) =_{\beta}$
 $(\lambda bb.bb)(\lambda b.bb) =_{\beta}$
 $(\lambda bb.bb)(\lambda b.bb) =_{\beta}$
 \dots
- (v) $(\lambda a.\lambda b.abc) p q =$
 $((\lambda a.(\lambda b.abc)) p) q =_{\beta}$
 $(\lambda b.pbc) q =_{\beta}$
 (pqc)
- (vi) $(\lambda p.((\lambda q.pqr)c))d =_{\beta}$
 $(\lambda p.pcr)d =_{\beta}$
 (dcr)
- (vii) $(\lambda ab.a)(\lambda p.p) =$
 $(\lambda a.(\lambda b.a))(\lambda p.p) =_{\beta}$
 $(\lambda b.(\lambda p.p))$
- (viii) NOT TRUE $=$
 $(\lambda p.(p \text{ FALSE TRUE})) \text{ TRUE} =_{\beta}$
 $\text{TRUE FALSE TRUE} =$
 $(\lambda x.(\lambda y.(x))) \text{ FALSE TRUE} =$
 $((\lambda x.(\lambda y.(x))) \text{ FALSE}) \text{ TRUE} =_{\beta}$
 $((\lambda y.\text{FALSE}) \text{ TRUE}) =_{\beta}$
 FALSE

- (ix) AND TRUE FALSE =
 $(\lambda p.(\lambda q.(p \ q \ \text{FALSE}))) \ \text{TRUE} \ \text{FALSE} =$
 $((\lambda p.(\lambda q.(p \ q \ \text{FALSE}))) \ \text{TRUE}) \ \text{FALSE} =_{\beta}$
 $(\lambda q.(\text{TRUE} \ q \ \text{FALSE})) \ \text{FALSE} =_{\beta}$
 TRUE FALSE FALSE =
 $(\lambda x.(\lambda y.(x))) \ \text{FALSE} \ \text{FALSE} =$
 $((\lambda y.\text{FALSE}) \ \text{FALSE}) =_{\beta}$
 FALSE
- (x) OR TRUE FALSE =
 $((\lambda p.(\lambda q.(p \ \text{TRUE} \ q))) \ \text{TRUE}) \ \text{FALSE} =_{\beta}$
 $(\lambda q.(\text{TRUE} \ \text{TRUE} \ q)) \ \text{FALSE} =_{\beta}$
 TRUE TRUE FALSE =
 $((\lambda x.(\lambda y.(x))) \ \text{TRUE}) \ \text{FALSE} =_{\beta}$
 $(\lambda y.\text{TRUE}) \ \text{FALSE} =_{\beta}$
 TRUE