

## Context-Free Languages: Questions

1. Consider the following grammar with start symbol S:

S  $\rightarrow$  AA  
A  $\rightarrow$  AAA  
A  $\rightarrow$  a  
A  $\rightarrow$  bA  
A  $\rightarrow$  Ab

- What strings can be produced by derivations of four or fewer steps?
  - Give at least four distinct derivations for the string babbab.
2. Construct context-free grammars which generate each of the following languages:
- $\{wcw^R \mid w \in \{a,b\}^*\}$
  - $\{ww^R \mid w \in \{a,b\}^*\}$
  - $\{w \in \{a,b\}^* \mid w = w^R\}$

3. Consider the following grammar with start symbol S:

S  $\rightarrow$  aB  
S  $\rightarrow$  bA  
A  $\rightarrow$  a  
A  $\rightarrow$  aS  
A  $\rightarrow$  BAA  
B  $\rightarrow$  b  
B  $\rightarrow$  bS  
B  $\rightarrow$  ABB

- Show that the string ababba belongs to this language.
  - Prove by induction that all strings in the language have equal numbers of a's and b's.
4. Consider the following grammar with start symbol S:

S  $\rightarrow$  PVP  
P  $\rightarrow$  N  
P  $\rightarrow$  AP  
N  $\rightarrow$  cheese  
N  $\rightarrow$  Jim  
V  $\rightarrow$  ate  
A  $\rightarrow$  green  
A  $\rightarrow$  big

Give a parse tree for the sentence "big Jim ate green cheese".

5. Consider the following grammar with start symbol S:

S  $\rightarrow$  SS  
S  $\rightarrow$  (S)  
S  $\rightarrow$   $\epsilon$

Show that this grammar is ambiguous by generating two different parse trees for the sentence (). Give an equivalent unambiguous grammar.

6. Consider the pushdown automaton with start state  $s$ , accepting state  $f$  and the following transitions:

$((s,a,\epsilon),(s,a))$   
 $((s,b,\epsilon),(s,b))$   
 $((s,a,\epsilon),(f,\epsilon))$   
 $((f,a,a),(f,\epsilon))$   
 $((f,a,b),(f,\epsilon))$   
 $((f,b,a),(f,\epsilon))$   
 $((f,b,b),(f,\epsilon))$

Indicate whether or not each of the following strings is accepted by this PDA:

- a. a
  - b. b
  - c. aa
  - d. ab
  - e. bb
  - f. aab
  - g. aaa
  - h. baa
  - i. bab
  - j. bbb
7. Construct a pushdown automaton to recognise the same language as that described by the grammar in question 5.
8. Give a context-free grammar which describes the same language as that recognised by the pushdown automaton in question 6.
9. Construct a pushdown automaton to recognise the following language:
- $$\{a^i b^j c^k \mid i, j, k \geq 0 \text{ and } i=j \text{ or } j=k\}$$
10. Use the pumping lemma to show that the following languages are not context-free:
- a.  $\{a^i b^j c^k \mid i < j < k\}$
  - b.  $\{w \in \{a,b,c\}^* \mid \text{the number of a's is equal to the maximum of the number of b's and the number of c's}\}$