Regular Languages: Answers

1. This language consists of those strings with 0 or more a's an then precisely one b at the end. This could be rewritten as a*b.
2.
    a. (a|b)*
    b. (a|b)*
    c. (a|b)*
    d. b* a (a|b)*
3.
    a. b* | b*ab* | b*ab*ab* | b*ab*ab*ab*
    b. (b*ab*ab*ab*)* | b*
    c. (b | ab | aab)* aaa (b | ba | baa)*
4.
    a. True
    b. True
    c. False
    d. False
5.
    a. The DFA is as follows:

State	Symbol	δ(State,Symbol)
0	a	0
0	b	0

where the start state is 0 and the accepting state is 0.

    b. The DFA is as follows:

State	Symbol	δ(State,Symbol)
0	a	0
0	b	0

where the start state is 0 and the accepting state is 0.

    c. The DFA is as follows:

State	Symbol	δ(State,Symbol)
0	a	0
0	b	0

where the start state is 0 and the accepting state is 0.

    d. The DFA is as follows:

State	Symbol	δ(State,Symbol)
0	a	1
0	b	0
1	a	1

1	b	1

where the start state is 0 and the accepting state is 1.

6.

a. The DFA is as follows:

State	Symbol	δ(State,Symbol)
0	a	1
0	b	0
1	a	2
1	b	1
2	a	3
2	b	2
3	a	4
3	b	3
4	a	4
4	b	4

where the start state is 0 and the accepting states are 0, 1, 2 and 3.

b. The DFA is as follows:

State	Symbol	δ(State,Symbol)
0	a	1
0	b	0
1	a	2
1	b	1
2	a	0
2	b	2

where the start state is 0 and the accepting state is 0.

c. The DFA is as follows:

State	Symbol	δ(State,Symbol)
0	a	1
0	b	0
1	a	2
1	b	0
2	a	3

2	b	0
3	a	6
3	b	4
4	a	5
4	b	4
5	a	3
5	b	4
6	a	6
6	b	6

where the start state is 0 and the accepting states are 3, 4 and 5.

7.

i. Yes
ii. Yes
iii. No
iv. Yes

i. Yes
ii. Yes
iii. Yes
iv. Yes
v. No

8.

a. The subset construction method gives the following:

State	Symbol	δ(State,Symbol)
0	a	{0,1}
{0,1}	a	{0,1}
{0,1}	b	1
1	b	1

where the start state is 0 and the accepting states are 0 and {0,1}.

b. The subset construction method gives the following:

State	Symbol	δ(State,Symbol)
0	a	1
1	b	{2,3}
{2,3}	a	{1,3}
{1,3}	a	1
{1,3}	b	{2,3}

where the start state is 0 and the accepting states are 0, {2,3} and {1,3}.

9. An NFA to recognise the same language is as follows:

State	Symbol	δ(State,Symbol)
0	a	0
0	b	0
0	a	1
1	a	1
1	b	1

   where the start state is 0 and the accepting state is 1.

   Converting this into an equivalent DFA gives the following:

State	Symbol	δ(State,Symbol)
0	a	1
0	b	0
1	a	1
1	b	1

   where the start state is 0 and the accepting state is 1.

10. In order to prove that a language is not regular using the pumping lemma, we must show that there are strings x, y and z such that $|y| > 0$ and xy^kz is an element of the language for all $k >= 0$. We assume that the language is regular, and obtain a contradiction.
    a. Let $s = a^kbba^k$. s must equal xyz, where x, y and z satisfy the conditions above. y must consist only of a's, so xy^2z cannot be a member of the language.
    b. Let $s = a^kba^kb$. s must equal xyz, where x, y and z satisfy the conditions above. y must consist only of a's, so xy^2z cannot be a member of the language.