

Regular Languages: Questions

1. What language is represented by the regular expression $((a^* a)b \mid b)$?
2. Rewrite each of the following regular expressions as simpler expressions describing the same language:
 - a. $\epsilon^* \mid a^* \mid b^* \mid (a|b)^*$
 - b. $((a^* b^*)^* (b^* a^*)^*)^*$
 - c. $(a^* b)^* \mid (b^* a)^*$
 - d. $(a|b)^* a (a|b)^*$
3. Let $\Sigma = \{a, b\}$. Write regular expressions to describe the following languages:
 - a. All strings in Σ^* with no more than three a's.
 - b. All strings in Σ^* with a number of a's divisible by three.
 - c. All strings in Σ^* with exactly one occurrence of the substring aaa.
4. Indicate whether each of the following is true or false:
 - a. $baa \in L(a^* b^* a^* b^*)$
 - b. $L(b^* a^*) \cap L(a^* b^*) = L(a^*) \cup L(b^*)$
 - c. $L(a^* b^*) \cap L(b^* c^*) = \emptyset$
 - d. $abcd \in L(a(cd)^* b)^*$
5. Construct deterministic finite automata to recognise each of the languages specified in question 2.
6. Construct deterministic finite automata to recognise each of the languages specified in question 3.
7.
 - a. Consider the following non-deterministic finite automaton:

State	Symbol	$\delta(\text{State}, \text{Symbol})$
0	a	0
0	a	1
1	b	1

where the start state is 0, and the accepting state is 0.

Indicate whether or not each of the following strings is accepted by this NFA:

- i. a
- ii. aa
- iii. aab
- iv. ϵ

b. Consider the following non-deterministic finite automaton:

State	Symbol	$\delta(\text{State}, \text{Symbol})$
0	a	1
1	b	2
1	b	3
2	a	3
3	a	1

where the start state is 0, and the accepting states are 0 and 3.

Indicate whether or not each of the following strings is accepted by this NFA:

- i. ϵ
- ii. ab
- iii. abab
- iv. aba
- v. abaa

8. Convert the NFAs in question 7 into equivalent DFAs which recognise the same language.

9. Consider the following regular grammar:

- $S \rightarrow aS$
- $S \rightarrow bS$
- $S \rightarrow aA$
- $S \rightarrow a$
- $A \rightarrow aA$
- $A \rightarrow bA$
- $A \rightarrow a$
- $A \rightarrow b$

Construct a finite state automaton which can be used to recognise the same language.

10. Use the pumping lemma to show that the following languages are not regular:

- a. $\{ww^r \mid w \in \{a,b\}^*\}$
- b. $\{ww \mid w \in \{a,b\}^*\}$