Turing Machines: Answers

1.
   a. The final configuration is: $(q_2,\underline{\#}\#abba\#)$
   b. The final configuration is: $(q_2,\underline{\#}\#baaaab\#)$
   c. For an arbitrary input string w in {a,b}*, starting in the initial configuration $(q_0,\underline{\#}w\#)$, this Turing machine halts with configuration $(q_2,\underline{\#}\#ww^R\#)$

2.
   a. The sequence of configurations is as follows:

      $(q_0,\underline{a}baa\#)$
      $(q_1,A\underline{b}aa\#)$
      $(q_1,Ab\underline{a}a\#)$
      $(q_1,Aba\underline{a}\#)$
      $(q_1,Abaa\underline{\#})$
      $(q_2,Aba\underline{a}\#)$
      $(q_3,Ab\underline{a}A\#)$
      $(q_3,A\underline{b}aA\#)$
      $(q_3,\underline{A}baA\#)$
      $(q_0,A\underline{b}aA\#)$
      $(q_1,AB\underline{a}A\#)$
      $(q_1,ABa\underline{A}\#)$
      $(q_2,AB\underline{a}A\#)$
      $(q_3,A\underline{B}AA\#)$
      $(q_0,AB\underline{A}A\#)$
      $(q_4,A\underline{B}AA\#)$
      $(q_4,\underline{A}bAA\#)$
      $(q_4,\underline{<}abAA\#)$
      $(q_5,\underline{a}bAA\#)$
      $(q_7,A\underline{b}AA\#)$
      $(q_7,Ab\underline{A}A\#)$
      $(q_8,A\underline{b}\#A\#)$
      $(q_8,\underline{A}b\#A\#)$
      $(q_5,A\underline{b}\#A\#)$
      $(q_6,AB\underline{\#}A\#)$
      $(q_6,AB\#\underline{A}\#)$
      Crash

   b. The sequence of configurations is as follows:

      $(q_0,\underline{a}bab\#)$
      $(q_1,A\underline{b}ab\#)$
      $(q_1,Ab\underline{a}b\#)$

$(q_1,Aba\underline{b}\#)$
$(q_1,Abab\underline{\#})$
$(q_2,Aba\underline{b}\#)$
$(q_3,Ab\underline{a}B\#)$
$(q_3,A\underline{b}aB\#)$
$(q_3,\underline{A}baB\#)$
$(q_0,A\underline{b}aB\#)$
$(q_1,AB\underline{a}B\#)$
$(q_1,ABa\underline{B}\#)$
$(q_2,AB\underline{a}B\#)$
$(q_3,A\underline{B}AB\#)$
$(q_0,AB\underline{A}B\#)$
$(q_4,A\underline{B}AB\#)$
$(q_4,\underline{A}bAB\#)$
$(q_4,\leq abAB\#)$
$(q_5,\underline{a}bAB\#)$
$(q_7,A\underline{b}AB\#)$
$(q_7,Ab\underline{A}B\#)$
$(q_8,A\underline{b}\#B\#)$
$(q_8,\underline{A}b\#B\#)$
$(q_5,A\underline{b}\#B\#)$
$(q_6,AB\underline{\#}B\#)$
$(q_6,AB\#\underline{B}\#)$
$(q_8,AB\underline{\#}\#\#)$
$(q_8,A\underline{B}\#\#\#)$
$(q_5,AB\underline{\#}\#\#)$
Accept

c. The language accepted by this Turing machine is $\{ww \mid w \in \{a,b\}^*\}$

3. A Turing machine to accept this language has the following transitions with start state q_0

State	Symbol	δ(State,Symbol)
q_0	a	$(q_1,\#,R)$
q_0	b	(q_2,b,R)
q_1	a	(q_1,a,R)
q_1	b	(q_1,b,R)
q_1	#	$(q_3,\#,L)$

q_2	b	(q_2,b,R)
q_2	#	$(q_2,\#,Y)$
q_3	b	$(q_4,\#,L)$
q_4	a	(q_4,a,L)
q_4	b	(q_4,b,L)
q_4	#	$(q_0,\#,R)$

4. A Turing machine to accept this language has the following transitions with start state q_0

State	Symbol	δ(State,Symbol)
q_0	a	(q_1,x,L)
q_0	b	(q_2,b,R)
q_0	x	(q_0,x,R)
q_0	#	$(q_0,\#,Y)$
q_1	b	(q_1,b,L)
q_1	x	(q_1,x,L)
q_1	$<$	$(q_3,<,R)$
q_2	a	(q_1,x,L)
q_2	b	(q_2,b,R)
q_2	x	(q_2,x,R)
q_3	a	(q_3,a,R)
q_3	x	(q_3,x,R)
q_3	b	(q_4,x,L)
q_4	a	(q_4,a,L)
q_4	x	(q_4,x,L)
q_4	$<$	$(q_0,<,R)$

5. A Turing machine to accept this language has the following transitions with start state q_0

State	Symbol	δ(State,Symbol)
q_0	a	$(q_1,\#,R)$
q_0	b	$(q_4,\#,R)$
q_0	#	$(q_0,\#,Y)$
q_1	a	(q_1,a,R)
q_1	b	(q_1,b,R)
q_1	#	$(q_2,\#,L)$
q_2	a	$(q_3,\#,L)$
q_2	#	$(q_2,\#,Y)$
q_3	a	(q_3,a,L)
q_3	b	(q_3,b,L)
q_3	#	$(q_0,\#,R)$
q_4	a	(q_4,a,R)
q_4	b	(q_4,b,R)
q_4	#	$(q_5,\#,L)$
q_5	b	$(q_3,\#,L)$
q_5	#	$(q_5,\#,Y)$

6.

a. A Turing machine to compute this function has the following transitions with start state q_0:

State	Symbol	δ(State,Symbol)
q_0	1	$(q_0,1,R)$
q_0	#	$(q_1,1,R)$
q_1	#	$(q_1,1,Y)$

b. A Turing machine to compute this function has the following transitions with start state q_0:

State	Symbol	δ(State,Symbol)
q_0	1	$(q_1,\#,R)$
q_0	x	$(q_0,1,R)$
q_0	#	$(q_0,\#,Y)$
q_1	1	$(q_1,1,R)$
q_1	x	(q_1,x,R)
q_1	#	(q_2,x,L)
q_2	1	$(q_2,1,L)$
q_2	x	(q_2,x,L)
q_2	#	$(q_0,1,R)$

c. A Turing machine to compute this function has the following transitions with start state q_0:

State	Symbol	δ(State,Symbol)
q_0	1	$(q_0,1,R)$
q_0	#	$(q_1,\#,L)$
q_1	1	$(q_2,\#,L)$
q_1	#	$(q_1,\#,Y)$
q_2	1	$(q_1,\#,L)$
q_2	#	$(q_1,1,Y)$

7. A Turing machine to compute this function has the following transitions with start state q_0:

State	Symbol	δ(State,Symbol)
q_0	a	$(q_1,\#,L)$
q_0	#	$(q_0,\#,Y)$
q_1	#	(q_2,a,R)

q_2	#	(q_0,#,R)

8. Let *M* be a linear bounded automaton with *q* states and *s* symbols. For an input string of length *n*, the maximum amount of tape which will be used will be of length *n*. There are therefore s^n possible strings which can appear on the tape. As there are *n* possible positions for the tape head and *q* possible states, there are qns^n possible configurations. If *M* has not halted within qns^n steps, it must be in a repeating configuration, and therefore looping. We can therefore decide after this number of steps whether or not the input string will be accepted or rejected. As linear-bounded automata are recognisers for context-sensitive languages, every context-sensitive language must be recursive.

9. A derivation of this sentence is as follows:

S
=> ABCS
=> ABCABCS
=> ABCABCABCS
=> ABCABCABCZ
=> ABACBCABCZ
=> AABCBCABCZ
=> AABBCCABCZ
=> AABBCACBCZ
=> AABBACCBCZ
=> AABABCCBCZ
=> AAABBCCBCZ
=> AAABBCBCCZ
=> AAABBBCCCZ
=> AAABBBCCZc
=> AAABBBCZcc
=> AAABBBYccc
=> AAABBYbccc
=> AAABYbbccc
=> AAAXbbbccc
=> AAXabbbccc
=> AXaabbbccc
=> Xaaabbbccc
=> aaabbbccc

The language generated by this grammar is: $\{a^n b^n c^n \mid n > 0\}$

10. Show that the following functions are primitive recursive:

d. A primitive rescursive definition for this function is as follows:

factorial(0) = succ(0)
factorial(succ(n)) = mult(succ(n),factorial(n))

As mult is primitive recursive, factorial is also primitive recursive.

e. A primitive recursive definition for this function is as follows:

gcd(m,0) = m
gcd(m,succ(n)) = gcd(succ(n),mod(m,succ(n)))

As mod is primitive recursive, gcd is primitive recursive.