Turing Machines: Questions

1. Consider the Turing machine with input alphabet {a,b}, start state q_0 and the following transitions:

State	Symbol	δ(State,Symbol)
q_0	#	(q_1,#,R)
q_1	a	(q_1,a,R)
q_1	b	(q_1,b,R)
q_1	#	(q_2,#,L)
q_2	a	(q_3,#,R)
q_2	b	(q_5,#,R)
q_2	#	(q_2,#,Y)
q_3	#	(q_4,a,R)
q_4	a	(q_4,a,R)
q_4	b	(q_4,b,R)
q_4	#	(q_7,a,L)
q_5	#	(q_6,b,R)
q_6	a	(q_6,a,R)
q_6	b	(q_6,b,R)
q_6	#	(q_7,b,L)
q_7	a	(q_7,a,L)
q_7	b	(q_7,b,L)
q_7	#	(q_2,#,L)

   a. What is the final configuration if the input is #ab#?
   b. What is the final configuration if the input is #baa#?
   c. Describe what the Turing machine does for an arbitrary input string in {a,b}*.

2. Consider the Turing machine with input alphabet {a,b}, start state q_0 and the following transitions:

State	Symbol	δ(State,Symbol)
q_0	a	(q_1,A,R)
q_0	b	(q_1,B,R)
q_0	A	(q_4,A,L)
q_0	B	(q_4,B,L)
q_0	#	(q_0,#,Y)
q_1	a	(q_1,a,R)
q_1	b	(q_1,b,R)
q_1	A	(q_2,A,L)
q_1	B	(q_2,B,L)
q_1	#	(q_2,#,L)
q_2	a	(q_3,A,L)
q_2	b	(q_3,B,L)
q_3	a	(q_3,a,L)
q_3	b	(q_3,b,L)
q_3	A	(q_0,A,R)
q_3	B	(q_0,B,R)
q_4	A	(q_4,a,L)
q_4	B	(q_4,b,L)
q_4	<	(q_5,<,R)
q_5	a	(q_7,A,R)
q_5	b	(q_6,B,R)
q_5	#	(q_5,#,Y)
q_6	a	(q_6,a,R)

q_6	b	(q_6,b,R)
q_6	B	(q_8,#,L)
q_6	#	(q_6,#,R)
q_7	a	(q_7,a,R)
q_7	b	(q_7,b,R)
q_7	A	(q_8,#,L)
q_7	#	(q_7,#,R)
q_8	a	(q_8,a,L)
q_8	b	(q_8,b,L)
q_8	A	(q_5,A,R)
q_8	B	(q_5,B,R)
q_8	#	(q_8,#,L)

a. Trace the execution of this Turing machine for the input string abaa.
b. Trace the execution of this Turing machine for the input string abab.
c. Describe the language accepted by this Turing machine.

3. Construct a Turing machine to accept the language $\{a^i b^j \mid i < j\}$
4. Construct a Turing machine to accept the language $\{w \in \{a,b\}^* \mid$ w contains the same number of a's as b's$\}$
5. Construct a Turing machine to accept the language $\{w \in \{a,b\}^* \mid w = w^R\}$
6. Construct Turing machines to compute the following functions (assume that the number x is in unary notation):
   a. $f(x) = x + 2$
   b. $f(x) = 2x$
   c. $f(x) = x \bmod 2$
7. Construct a Turing machine to implement a "shift machine" as described in the lecture notes.
8. Show that every context-sensitive language is recursive (hint: how long can derivations be?)
9. Consider the following unrestricted grammar:

S -> ABCS
S -> Z
CA -> AC
BA -> AB
CB -> BC
CZ -> Zc

CZ -> Yc
BY -> Yb
BY -> Xb
AX -> Xa
X -> ε

Give a derivation of the sentence aaabbbccc. What is the language generated by this grammar?

10. Show that the following functions are primitive recursive:
    a. factorial(n) = n!
    b. gcd(m,n) = the greatest common divisor of m and n