Solutions to Exercise 9

Q1

- First, the prey is the same in both systems because the growth rate per capita for the prey in both system is the same, 0.3. This is the only coefficient that identifies the prey alone, so the prey is the same in both systems.

- Interaction
  - The system on the left has an interaction coefficient 0.1, which means that the predator \( y \) does not affect too much the prey population.
  - The opposite happens with the system on the right, where 3, so the predator \( y \) on the right system hunts much more prey than the \( y \) on the left system. This suggests \( y \) in the left system is the boas and the \( y \) in the right system refers to bobcats.

- From the second equations in both systems we draw similar conclusions. The decay coefficient 0.1 is smaller than 2, which means that the predator in the left system can live longer times without food than the predator in the right system. This agrees with the fact that boas are cold blooded and more energy efficient in warm weather than the warm blooded bobcats.

- Finally, the interaction coefficient 2 means that the population of the predator \( y \) gains a lot more form each prey than the population of the right hand predator, which has a smaller coefficient 0.1. Again this suggests that left are boas and right are bobcats.

Q2 (i)

The differential equations are given by:

\[
\frac{dx}{dt} = -\alpha_{yx} x + \alpha_{xy} y \quad (1) \text{ and } \frac{dy}{dt} = +\alpha_{yx} x - \alpha_{xy} y \quad (2)
\]

LHS of these equations gives the rate of change of the market share of the brand with time and RHS shows the coupling between the two Des.

If we add these DEs we get \( \frac{dx}{dt} + \frac{dy}{dt} = 0 \),

which on integrating wrt time gives Answer (ii) \( x(t)+y(t)=1 \) (3)

Q2 (iii)

Setting (1) =0 above gives

\[
\alpha_{yx} \bar{x} = \alpha_{xy} \bar{y} \quad (4)
\]

But from (3) \( \bar{x} + \bar{y} = 1 \Rightarrow \bar{y} = 1 - \bar{x} \)

Hence (4) becomes

\[
\alpha_{yx} \bar{x} = \alpha_{xy} (1 - \bar{x})
\]

or

\[
\bar{x} = \frac{\alpha_{xy}}{\alpha_{xy}+\alpha_{yx}} \quad \text{QED}
\]

Similar proof for \( \bar{y} \)

Q2 (iv)

Equilibrium values are given by \( \bar{x} = \frac{1}{3}, \bar{y} = \frac{2}{3} \)