Solutions to Exercise 5

Q2 $M_{n+1} = M_n + 2M_{n-1}$ matrix equation is

$$
\begin{bmatrix} M_{n+1} \\ M_n \end{bmatrix} =
\begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}
\begin{bmatrix} M_n \\ M_{n-1} \end{bmatrix}, \quad \lambda_{\text{max}} = 2.414 \text{ so increases.}
$$

Diagram

<table>
<thead>
<tr>
<th>Season</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Pair</td>
<td>$\uparrow$</td>
<td>$\rightarrow$</td>
<td>$\rightarrow$</td>
<td>$\rightarrow$</td>
<td>$\rightarrow$</td>
</tr>
<tr>
<td>Total Number of Pairs:</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>5</td>
<td>11</td>
</tr>
</tbody>
</table>

Q3 As $\lambda_1 > 1$, we can say that the populations of both the lions and wildebeest will increase with time. In the long term there will be 100 wildebeest for every 0.467 lion or 214 wildebeest per lion.

Q4 Deep in the redwood forest of California, dusky-footed wood rats provide up to 80% of the diet for the spotted owl, the main predator of the wood rat.

- In the absence of wood rats, the owl population would decay by 50% each year.
- If rats are plentiful, then the owl population from year to year will increase by 40% of the total rat population.
- In the absence of owls, the population of the rats will grow by 10% per year.
- The owls hunt the rats, so if owls are present, the rat population will fall by 10% of the total owl population.

If $R_n, R_{n+1}$, and $O_n, O_{n+1}$ denote the rat and owl population in years $n$ and $n+1$ and

$$
R_{n+1} = aR_n + bO_n \\
O_{n+1} = cR_n + dO_n
$$

Determine the values of $a, b, c, d$.

$$
R_{n+1} = 1.1R_n - 0.10O_n \\
O_{n+1} = 0.4R_n + 0.5O_n
$$