Time Series Analysis: Session Learning Objectives

After this session, you will be able to

1. Analyse and explain the main components of a time series
2. Employ graphical procedures to examine a time series for possible trends
3. Describe the trend in the series
   - Using a moving average
   - By fitting a line to the trend
4. Be aware of the upsides and downsides of moving averages and linear regression
5. Employ the trend to do simple forms of forecasting

General approach to analysis (1/2)

We elaborate here on the general steps seen in the previous session to analysing time series data:

1. **Step 1**
   Plot the data with time on x-axis

2. **Step 2**
   Study the pattern over time
   - Is there any trend evident in the series?
   - Does it manifest any seasonality effect?
   - Are there any long term cycles?
   - Does data change sharply and if so, how to explain?
   - Any outliers, (data differing widely from general pattern?)
   - Can these outliers be explained?
Step 3
Outline clearly and succinctly the point of analysing the time series

Step 4
Paying attention to the objectives:
– Analyse the data using descriptive analyses first,
– Follow up with other more sophisticated forms of analyses

Step 5
Report the key results, main findings and conclusions of the analysis

Main Components of a Time Series (1/6)

Level Component
Long-term average - i.e. data fluctuations around constant mean

Trend
The general direction in which the series is running during a long period (i.e. long-term increase or decrease)

Seasonal effects
Short-term fluctuations that occur regularly - often associated with (or influenced by) months or quarters

Cyclical effects
Long-term fluctuations occurring regularly in the series, mostly rises/falls of no fixed periodicity, longer than 2 years

Residual
Any remaining after the above have been accounted for, i.e. unexplained (random) components of variation
Main Components of a Time Series (2/6)

(a) Level  
(b) Trend  
(c) Seasonality  
(d) Cycle

Above we show four patterns in time series

Main Components of a Time Series (3/6)

**Figure 2.10:** A Figure Showing Monthly Housing Sales

- The monthly housing sales above show
  - Strong seasonality within each year,
  - Some evidence of strong cyclicity (period 6 – 10 years)
  - There is no apparent trend in the data over this period.
Results from the Chicago CBOT market for 100 straight trading days in 1981 shows:
- There is no seasonality,
- An obvious downward trend?
- Longer series could show this is part of a long cycle, but over 100 days it appears to be a trend.

Australian monthly electricity production data (above):
- Shows a strongly upward/increasing trend,
- Manifests evidence of strong seasonality.
- But no sign of cyclicality (i.e. long-term fluctuations).
Examining Trends (1/2)

- Assume time series data has at least 1 systematic pattern
  - Most common patterns are trends and seasonality.
  - **Trends:**
    - Trends are generally linear or quadratic.
    - Can often find with moving averages or regression analysis
  - **Seasonality:**
    - This is a trend repeating itself systematically over time.

- Assume also that data exhibits enough of a random process to be hard to identify systematic patterns.
  - Time series analysis techniques often employ some type of filter to the data in order to dampen the error.
  - Other potential patterns have to do with lingering effects of earlier observations or earlier random errors.

**FIGURE 2.13:** A Figure Showing Dow Jones Index Daily Data

- Daily change in the Dow Jones index data (above) shows:
  - No trend, seasonality or cyclic behaviour.
- The random fluctuations in the data:
  - Do not appear to be very predictable,
  - Also no strong patterns that would help with developing a forecasting model.
Examining Trends (2/2)

The following slides include:
- Identifying if there appears to be a trend
- Describing and examining the trend using
  - Moving averages
  - Fitting a straight line to the data
- Simple approaches to forecasts, and likely dangers

Nóta Beag
Cyclical effects will not be covered since they can be difficult to identify and need long series

Example 2.5: Trends

Data\(^4\) shows the number of unemployed school leavers in the UK (in 000s)

<table>
<thead>
<tr>
<th>Year</th>
<th>Jan to Mar</th>
<th>Apr to Jun</th>
<th>Jul to Sep</th>
<th>Oct to Dec</th>
</tr>
</thead>
<tbody>
<tr>
<td>1979</td>
<td>22</td>
<td>12</td>
<td>110</td>
<td>31</td>
</tr>
<tr>
<td>1980</td>
<td>21</td>
<td>26</td>
<td>150</td>
<td>70</td>
</tr>
<tr>
<td>1981</td>
<td>50</td>
<td>36</td>
<td>146</td>
<td>110</td>
</tr>
</tbody>
</table>

\(^4\)Source: Employment Gazette
Example 2.5: Putting the Data in List Format

<table>
<thead>
<tr>
<th>Year</th>
<th>Qtrs</th>
<th>Leavers</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1979</td>
<td>22</td>
</tr>
<tr>
<td>1</td>
<td>1979.25</td>
<td>12</td>
</tr>
<tr>
<td>1</td>
<td>1979.5</td>
<td>110</td>
</tr>
<tr>
<td>1</td>
<td>1979.75</td>
<td>31</td>
</tr>
<tr>
<td>2</td>
<td>1980</td>
<td>21</td>
</tr>
<tr>
<td>2</td>
<td>1980.25</td>
<td>26</td>
</tr>
<tr>
<td>2</td>
<td>1980.5</td>
<td>150</td>
</tr>
<tr>
<td>2</td>
<td>1980.75</td>
<td>70</td>
</tr>
<tr>
<td>3</td>
<td>1981</td>
<td>50</td>
</tr>
<tr>
<td>3</td>
<td>1981.25</td>
<td>36</td>
</tr>
<tr>
<td>3</td>
<td>1981.5</td>
<td>146</td>
</tr>
<tr>
<td>3</td>
<td>1981.75</td>
<td>110</td>
</tr>
</tbody>
</table>

**TABLE 2.5: Quarterly Data of UK School Leavers**

- Decimals for data only to approximate time points
- Graph shows clear seasonality & trend effects

### Estimating Trend I - Moving Average

Ignore seasonality for now & concentrate on estimating trend . . .

**Purpose:**
- To ‘smooth’ out local variation and possibly seasonal effects

**Method:**
- Replace each observation by a weighted average of observations around (& including) the particular observation

**Results will depend on:**
- number of observations used
- weights used
Suppose we have the series:

\[ Y_1, Y_2, Y_3, \ldots, Y_n \]

Then the \( i \)th value of the **Central Moving Average** is:

\[ Y_i^* = \frac{1}{5} Y_{i-2} + \frac{1}{4} Y_{i-1} + \frac{1}{4} Y_i + \frac{1}{4} Y_{i+1} + \frac{1}{5} Y_{i+2} \]

This sliding window of size 5 is centred on point \( i \).

A sliding window of size 4 has weights \( \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4} \).

This sliding window gives a **Simple Moving Average**.

**Points to Note on Weighted Moving Averages**

- Weights for moving averages must always add to 1.
- To find a moving average, must decide on order, i.e. how many to average? Quarterly can use order=4.
- These are reported at quarter end but refer to full quarter.
- For moving average of order \( r \):
  - For odd \( r \), moving average values cannot be defined for the first \( \frac{r-1}{2} \) and last \( \frac{r-1}{2} \) series values.
  - For even \( r \), moving average values lie midway between observation points.
- Better to have averages coinciding with observed data.
- For data not centred around the mean, simple MA lags or leads the latest datum point by half the sample width.
- This could have the effect of biasing the data in some way.
How Many Readings to Include in a Moving Average?

- No firm answer, but there is a trade-off to consider.
- Suppose the mean of the underlying process remains stable:
  - If too few data points, then the MA exhibits more variability than if we include a larger number of data points.
  - In that sense, get more stability from including more points.
- Suppose there is an unanticipated change in the mean of the underlying process:
  - If too few data points, MA will track the changed process more closely than if more of data points.
  - In that case, get more responsiveness by including fewer points.

Possible Action to Take When $r$ is Even

- If data are in quarters could use a 4-point MA first with equal weights \( \left( \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4} \right) \)
- Then use a further MA of order 2 on the first computed MA, again with equal weights, i.e. \( \frac{1}{4}, \frac{1}{4} \)
- This is equivalent to the expression before i.e. using weights \( \frac{1}{8}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{8} \) on the original data.
- This is a so-called 5-point weighted MA (or a 2X4 MA) with weights \( \left( \frac{1}{8}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{8} \right) \)
- Can see this from last column of example on the next page
Example 2.5(Revisited): Calculating Moving Averages

<table>
<thead>
<tr>
<th>Year</th>
<th>Quarter</th>
<th>Value of 01/03</th>
<th>3-Point Mean</th>
<th>4-Point MA</th>
<th>5-Point WMA</th>
</tr>
</thead>
<tbody>
<tr>
<td>1979</td>
<td>1</td>
<td>48</td>
<td>48</td>
<td>43.75</td>
<td>43.63</td>
</tr>
<tr>
<td>1979.25</td>
<td>12</td>
<td>51</td>
<td>54</td>
<td>43.5</td>
<td>45.25</td>
</tr>
<tr>
<td>1979.5</td>
<td>110</td>
<td>43.75</td>
<td>43.75</td>
<td>43.75</td>
<td>43.75</td>
</tr>
<tr>
<td>1979.75</td>
<td>31</td>
<td>54</td>
<td>54</td>
<td>43.5</td>
<td>45.25</td>
</tr>
<tr>
<td>1980</td>
<td>2</td>
<td>26</td>
<td>26</td>
<td>47</td>
<td>52</td>
</tr>
<tr>
<td>1980.25</td>
<td>26</td>
<td>65.7</td>
<td>57</td>
<td>61.88</td>
<td></td>
</tr>
<tr>
<td>1980.5</td>
<td>150</td>
<td>82</td>
<td>82</td>
<td>66.75</td>
<td>70.38</td>
</tr>
<tr>
<td>1980.75</td>
<td>70</td>
<td>90</td>
<td>90</td>
<td>74</td>
<td>75.25</td>
</tr>
<tr>
<td>1981</td>
<td>3</td>
<td>50</td>
<td>52</td>
<td>76.5</td>
<td>76</td>
</tr>
<tr>
<td>1981.25</td>
<td>36</td>
<td>77.3</td>
<td>75.5</td>
<td>80.5</td>
<td></td>
</tr>
<tr>
<td>1981.5</td>
<td>146</td>
<td>97.3</td>
<td>85.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1981.75</td>
<td>110</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Table 2.6:** Calculations on Table 2.5 data

- Verify a few of the values in the last 2 columns...

Example 2.5: Time Series & Fitted Trend

- The increasing trend is now clearly visible.
Exponential Moving Average (EWMA)/ Exponential Smoothing

- Probably most-used ad-hoc method to smooth time series

- **Advantages**
  - SMAs weight past values equally, EWMA assign explicitly shrinking weights over time.
  - Approximates a simple ‘memory’ for recent & older values.
  - ‘Simple exponential smoothing’ (SES) is simplest exponentially smoothing method.
  - Suitable for forecasting data with no obvious pattern - used to remove random effects mainly.

**Figure 2.14: Saudi Oil Data**

- No clear trend or seasonality in Fig 2.14 tho' mean may be slowly time-varying.
- **Plot from R code**

```r
oildat <- window(oil,start=1996,end=2007)
```
EWMA (/3): Example 2.6 Forecasting Saudi Oil

- Equation forecasts next value based on previous ones:
  \[ Y_{T+1/T} = \alpha y_T + \alpha(1-\alpha)y_{T-1} + \alpha(1-\alpha)^2y_{T-2} + \ldots \]

Exponential Moving Average (EWMA) (/4)

<table>
<thead>
<tr>
<th>( \alpha )</th>
<th>( 1-\alpha )</th>
<th>( (1-\alpha)^2 )</th>
<th>( (1-\alpha)^3 )</th>
<th>( (1-\alpha)^4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.9</td>
<td>0.1</td>
<td>0.01</td>
<td>0.001</td>
<td>0.0001</td>
</tr>
<tr>
<td>0.5</td>
<td>0.5</td>
<td>0.25</td>
<td>0.125</td>
<td>0.0625</td>
</tr>
<tr>
<td>0.1</td>
<td>0.9</td>
<td>0.81</td>
<td>0.729</td>
<td>0.6561</td>
</tr>
</tbody>
</table>

**Table 2.7:** Choosing Optimal \( \alpha \): Damping of Past Values

- Small (smoothing factor) \( \alpha \approx 0 \), past values get more weight.
- Large \( \alpha \approx 1 \), recent observations are more important.
- Or (from Table 2.7) smaller values of \( \alpha \) cause past values to damp out slower & vice versa.
Advantages & Disadvantages of Moving Averages

**Advantages**

- Simple, flexible, fairly intuitive and easily computed method.
- If the period of MAs coincides with the period of cyclic fluctuations in the data, latter are automatically eliminated.
- The method is ideal for using to determine seasonal, cyclic and irregular variations beside the trend values.

**Disadvantages**

- No trend values for some time values.
- SMA not a math function so not very helpful in forecasting.
- Tho EWMA has some applicability here (see below).
- Selecting the order of MA is also difficult.
- For non-linear trend values obtained can be biased in one/other direction.
Another way: find an equation giving the ‘best fit’ to the trend (i.e. minimise errors btw value and line).

The simplest is a straight line, i.e.

\[ Y = a + bX \]

- \( a \) is the intercept with the Y axis 
- \( b \) is the slope of the best fit line

Formulae for \( a, b \) which minimise the squared distances of each data point from the line, are on the last slide

These are an example of so-called Least Squares Method

Note:
- Assume for example a linear trend in the data
- Where this is not the case we have a different formula

Ex 2.5: School Leavers - Least Squares Regression

Intercept \( a \) is shown on the graph.

The slope \( b \) is given by \( b = \frac{\sum Y}{\sum X} \)

The equation of the line can be found to be

\[ Y = 19.4 + 7.07X \]

Note: \( X \) is the time elapsed (in quarters) since 1979.
Forecasting - Simple Approach

- Return to ‘best fit’ line to the data, i.e. \( Y = 19.4 + 7.07X \)
- Future value of \( X = X_i, Q13 \), can find a value \( Y = Y_i \)

\[
X_i = 19.4 + 7.07 \times 13 = 111.3
\]

- However, this approach has many dangers and shouldn’t do it without recognising these:
  1. Line fitting is part of a wider topic called ‘Regression’
  2. Assume repeat observations taken in time are independent. This is rarely the case.
  3. Also assume prediction worsens as \( X \) used moves away from the known time values.
  4. Shouldn’t forecast without knowing of many statistical & other practical issues involved.

Advantages & Disadvantages of (Linear) Least Squares Regression

- **Advantages**
  - The method gives trend values for entire time period without omission.
  - Trend line is a functional relationship between values & time, so method can be used to forecast future trend.
  - This is a completely objective method.

- **Disadvantages**
  - Certain amount of calculations required making it seem tedious and complicated.
  - Future forecasts by this method based only on trend values; ignore seasonal, cyclical or irregular variations.
  - If even a single item is added to the series a new equation has to be formed.
Calculations for ‘Best Fit’ Line

- Let the equation of line be:
  \[ Y = b_0 + b_1 X \] (2.1)

  where \( Y_i \) are the to quarterly time values, incrementing by 1 Quarter.

- Then
  \[ b_1 = \frac{\sum_{i=1}^{n} (X_i - \bar{X})(Y_i - \bar{Y})}{\sum_{i=1}^{n} (X_i - \bar{X})^2} \]

- and
  \[ b_0 = \bar{Y} - b_1 \times \bar{X} \]