Time Series Analysis: Session Learning Objectives

After this session, you will be able to

- Explain the form of two typical time series models:
  - Additive model
  - Multiplicative model
- Estimate the Trend/ Seasonal/ Cyclical effects in a series
- Describe the procedure for extracting these effects from a series
- Describe the use of such extracted patterns of a series

Recall Classical Decomposition

- Components of a Time Series:
  - Trend($T_t$): Long term movement in the mean
  - Seasonal variation($S_t$): Short-term fluctuations, usually assumed to be within a year hereafter in these notes
  - Cycle($C_t$): these are long-term cyclical patterns e.g. sun-spots, business cycles (these are not considered here)
  - Residuals ($R_t$): random/other unexplained variation components
- For seasonality, need to decide whether the series is better represented by an:
  - Additive model, i.e. $y_t = T_t + S_t + R_t$
  - Multiplicative model, i.e. $y_t = T_t \times S_t \times R_t$
Example 2.6: A Simple Example

- Fig 2.14 shows a typical real life time series.
- Notice the slight increase in variation over time.
- As will be seen, series like this require using a multiplicative model.

**Figure 2.14**: Sri Lankan Tourist Arrivals

Additive or Multiplicative Model?

- **Additive model**
  - Suits if the size of seasonal fluctuations (or variation around the trend \( T_t \)) doesn’t vary with the time series level\(^2\).
  - Means the seasonality is the same (roughly constant) in same period over different years (does not depend on level)

- **Multiplicative model**
  - Suits if the variation in seasonal pattern or that around the trend \( T_t \) does vary with the time series level.
  - With economic time series, such models are common.
  - Sometimes seasonal effect is a proportion of underlying trend value, e.g. in previous slide, they increase with trend

\(^2\)Level can roughly be described as long-term mean of a time series.
Example 2.7: A More Complex Example

- Figure 2.15 shows Electrical Equipment Orders.
- The trend component is shown in red, raw data in grey.
- Next slide shows an additive decomposition of this data.

![Figure 2.15: Some Electrical Equipment Order Data](image)

Example 2.7: A More Complex Example (/2)

![Figure 2.16: Additive Decomposition of Figure 2.15](image)
Example 2.7: A More Complex Example (/3)

**Figure 2.17**: Original Data & Seasonal from Figure 2.16
- Figure 2.16 enlarged in Figure 2.17
- Data in (b), (c) & (d) (next slide) sum to original data in (a).
- Note (b) varies slowly over time, so any 2 years in a row have similar patterns, but years far apart may have different ones.

Example 2.7: A More Complex Example (/4)

**Figure 2.18**: Trend & Remainder from Figure 2.16
- (d) is remainder when seasonality, trend taken from the data.
- The bars at right show the relative scales of components.
- Each same length but sizes vary as plots are on different scales.
Aside: Estimating the Seasonal Effect

- Recall that seasonal effects occur when the series repeats systematically in short time periods (often within a year).
- A de-seasonalised series shows the pattern of change over time with all seasonal effects removed.
  - allows direct comparisons between time points in this series, unaffected by seasonal changes.
- First de-trend the series by finding either:
  \[ D_i = Y_i - T_i \] additive model or \[ D_i = Y_i / T_i \] multiplicative model.
- Means of detrended values \( D_i \) are scaled so seasonal mean:
  - averages to zero for an additive model, or
  - averages to 1 for a multiplicative model.
- Seasonal means are often called Seasonal Indices (SI) and expressed as a percentage value.

Seasonally-Adjusted (SA) Data

- If seasonality is not the main focus, SA data can be of use.
- Example: Monthly Unemployment Data
  - Usually SA to show variation due to underlying state of the economy rather than seasonal variation.
  - Seasonal Variation: Jobless rises from school leavers seeking jobs.
  - Non-Seasonal Variation: Jobless rises due to large employers laying off workers.
- Most studying jobless data focus on non-seasonal variation.
- So employment (and other economic) data are usually SA.
- SA series contain remainder & trend components.
- So they not ‘smooth’ & ‘downturns’/‘upturns’ can mislead.
- If turning points in series are the focus & must interpret series changes, better to use trend not SA data.
Example 2.7: A More Complex Example (/5)

- If the seasonal part is removed from the raw data, resulting data are **Seasonally Adjusted**.
- Figure 2.19 shows seasonally adjusted electrical equipment orders.

![Figure 2.19: Some Electrical Equipment Order Data](image)

**TABLE 2.7:** Quarterly Irish Registered Births data (1982 to 92)

<table>
<thead>
<tr>
<th>Year\Quarter</th>
<th>Q1</th>
<th>Q2</th>
<th>Q3</th>
<th>Q4</th>
<th>Totals</th>
</tr>
</thead>
<tbody>
<tr>
<td>1982</td>
<td>17474</td>
<td>18475</td>
<td>18258</td>
<td>16726</td>
<td>70933</td>
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<tr>
<td>1983</td>
<td>17087</td>
<td>17260</td>
<td>17024</td>
<td>15444</td>
<td>66815</td>
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<tr>
<td>1984</td>
<td>16424</td>
<td>16218</td>
<td>16155</td>
<td>15440</td>
<td>64237</td>
</tr>
<tr>
<td>1985</td>
<td>15913</td>
<td>16012</td>
<td>15683</td>
<td>14642</td>
<td>62250</td>
</tr>
<tr>
<td>1986</td>
<td>15094</td>
<td>16663</td>
<td>15164</td>
<td>14504</td>
<td>61425</td>
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<tr>
<td>1987</td>
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<td>15406</td>
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<td>13530</td>
<td>58864</td>
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<tr>
<td>1988</td>
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<td>14426</td>
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<td>12050</td>
<td>54300</td>
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<td>1989</td>
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<td>13947</td>
<td>13158</td>
<td>11421</td>
<td>51659</td>
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<tr>
<td>1990</td>
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<td>13989</td>
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<td>12359</td>
<td>52954</td>
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<td>1991</td>
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<td>13620</td>
<td>12256</td>
<td>52690</td>
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<tr>
<td>1992</td>
<td>13565</td>
<td>13399</td>
<td>13150</td>
<td>11470</td>
<td>51584</td>
</tr>
</tbody>
</table>

- Data seems to change wildly between quarters.
- Totals column (right) shows trend seems to be downwards.
An Intro to Time Series Analysis
Trends in Time Series
Time Series Decomposition
Advanced Models of Univariate Time series
Forecasting in Univariate Time series
Summary of Time Series

Example 2.8: Births Data 1982-92 (/2)

Figure 2.20: Plot of Table 2.7 Births Data
- Shows an annual seasonality – not obvious in raw data.
- Annual Q4 data (Oct-Dec) lower than other 3 quarters.
- Fluctuations & general trends commented on are obvious.

Ex 2.8 Births 1982-92 (/3): Calculations

Table 2.8: Calculations on Table 2.7
- Trend calculated as $2 \times -1$ pt. MAs (n.b. first 2 values missing)
- Detrended values are Birth values/these Trend values
- Seasonal Indices (SI) are means of q'lies, $D_i$ (not all shown)
- SA or deseasonalised numbers are Births/SI values.
Ex 2.8 Births 1982-92 (/4): Trend Component

**Figure 2.21**: Irish Registered Births data Trend (1985 to 92)
- Compare main to that calculated using a 7 point MA (inset)
- Main is smoother than 7 pt MA which is based on an equal weighting of 7 data points rather than a WMA.

Ex 2.8 Births 1982-92 (/5): Seasonal Component

**Figure 2.22**: Irish Registered Births Data Seasonality (1985 to 92)
- Seasonal Indices shown represent raw data’s seasonal part
- Large fall in values is obvious on the scale shown.
Ex 2.8 Births (/6): Raw & Seasonally Adjusted

**Figure 2.23:** Raw & Seasonally Adjusted (SA) Births Data (1985 to 92)
- Shows SA smoother than raw, rougher than trend series.
  - SA more important than raw due to its more scientific basis – not as much subjectivity as trend estimation
  - SA level from 1989 – except for '89 Q4, '92 Q4 (maybe macroeconomic circumstances)

Example 2.9: Air Passengers

**Figure 2.24:** Monthly International passengers (in Millions) 1949-60
- ‘Seasonal’ package in R facilitates seasonal adjustment.
  - Use R Box & Jenkins dataset ‘AirPassengers’ with ‘Seasonal’
  - Data shows a high degree of seasonality, as seen in fig. 2.24.
Ex 2.9 Air Passengers (/2): Raw & SA

**Figure 2.25**: Air Passengers: Raw & Seasonally Adjusted
- Post-SA, examine changes & attribute these to fundamentals.
  - Fig. 2.25 shows both raw & SA series
  - Note that the SA series in red is much smoother than the original
  - What sort of decomposition do you think was done here? Why?

Ex 2.10: CO2 Data: ‘Lazy’ Series Decomposition

**Figure 2.26**: CO2 Series ‘Lazy’ Decomposition in R
- Full decomposition can also be done with `decomp()` in R.